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**A CRUISE CONTROL INSTRUMENT  
FOR TACTICAL TURBOJET AIRCRAFT**

**John E. Draim  
and  
Charles W. Meyrick**













A CRUISE CONTROL INSTRUMENT FOR TACTICAL  
TURBOJET AIRCRAFT

by

John E. Draim, Lieutenant, U.S.N.  
B.S. U.S. Naval P.G. School, 1955

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SUBMITTED IN PARTIAL FULFILLMENT OF THE  
REQUIREMENTS FOR THE DEGREE OF  
MASTER OF SCIENCE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

1956

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# A CRUISE CONTROL INSTRUMENT FOR TACTICAL TURBOJET AIRCRAFT

by

John E. Draim

Charles W. Meyrick

Submitted to the Department of Aeronautical Engineering on  
May 23, 1956 in partial fulfillment of the requirements for the  
degree of Master of Science.

## ABSTRACT

This thesis presents a proposal for a cruise control instrument for tactical turbojet aircraft with thrust-limited cruise characteristics and is based upon an investigation of cruise control techniques required to achieve maximum range or endurance.

Emphasis is placed upon the maximum range problem. Schemes of instrumentation and presentation are suggested that will provide the pilot with information which will enable him to achieve maximum usefulness from available fuel when flying certain flight profiles. Means for predicting and indicating the range and endurance remaining are also suggested.

The accuracy of the proposed instrument hinges primarily on more accurate aircraft instruments than are presently available for measuring fuel quantity, fuel flow, and ambient temperature.

A quantity required in the instrument for prediction purposes is temperature at altitudes other than the aircraft's altitude. An analytical expression for relating atmospheric temperature ratio with pressure ratio is developed, and a means of instrumenting the expression is proposed which involves inputs of tropopause altitude and isothermal layer temperature.

Thesis Supervisor: Yao Tzu Li

Title: Associate Professor of  
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The graduate work for which this thesis is a partial requirement was performed while the authors were assigned to the U.S. Naval Administrative Unit, Massachusetts Institute of Technology.



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## OBJECT

The object of this thesis is to study possible ways of building a cruise control instrument for tactical turbojet aircraft with thrust-limited cruise characteristics and then to make a preliminary proposal for the design of such an instrument.

The scope of this proposal will include: design specifications; instrumentation requirements; design of required display and control panels; and block diagrams containing some detailed design features of selected components of the system. The overall instrumentation problem will be examined and the range portion of the problem investigated in some detail.



## CHAPTER 1

### INTRODUCTION

Since the advent of the turbojet engine for aircraft, the effectiveness of tactical aircraft has, to a large degree, become dependent on proper cruise control techniques. To achieve maximum effectiveness from a present day jet aircraft the pilot must be aware of and take advantage of the widely varying fuel consumption rates of the turbojet engine under different cruise conditions. To get the maximum benefit from a tactical jet aircraft's fuel using the instruments presently found in such aircraft requires too much of the pilot's time and attention. Even then the pilot is not likely to follow the optimum flight path. The result is that such jet aircraft are usually not operated efficiently, thus cutting down their effectiveness.

Thus, there are two prime purposes of the proposed instrument. The first is to indicate to the pilot the best speed and altitude that should be flown in order to achieve the maximum use of his fuel for the mode of flight he desires. The second is to indicate to the pilot the range and endurance that can be expected if the aircraft is operated at the indicated speed and altitude.

It is expected that through the use of an instrument such as the one proposed here it will be possible for turbojet aircraft to be operated at longer ranges and greater endurances than have previously been possible from an operational standpoint, and to do so with very little extra effort on the part of the pilot. Also by employing such an instrument it should be possible to operate turbojet aircraft safely with smaller end of flight fuel reserves.

The instrument proposed in this report is for tactical turbojet aircraft with thrust limited cruise characteristics.

Due to time limitations and the scope of the problem a complete investigation of the proposed instrumentation problem has not been attempted. However, the complete problem has been outlined and the range portion of the problem investigated in some detail to indicate the philosophy and method of attack to be used in solving the complete problem.

## CHAPTER 2

### SPECIFICATIONS FOR CRUISE CONTROL INSTRUMENT

The specifications outlined in this chapter are those considered necessary in order that the instrument fulfill its mission of permitting the pilot of a turbojet aircraft to derive more effective use of available fuel. The specifications outlined here would apply to turbojet aircraft with Mach-limited cruise characteristics, or thrust-limited cruise characteristics.

The specified outputs of this instrument have been limited to only those considered to be essential, in order to limit the size, weight and complexity of the instrument. It is believed that additional information, particularly pre-flight planning type information, could better be supplied by a computer on the ground.

#### I Flight Modes

Means should be provided so that the pilot can select the flight mode for which the instrument will supply flight information. These modes should be:

- A. Best climb for maximum range or endurance.
- B. Best cruise for maximum range or endurance.
- C. Best cruise for maximum range or endurance at constant altitude.
- D. Best range descent using an idle throttle setting.



## II Flight Instrument Outputs

The following are the flight instrument outputs that should be presented to the pilot for the different flight modes he may select:

- A. When the best climb mode for maximum range or maximum endurance is selected the instrument should give a continuous indication of the best climbing speed to be employed and in addition should indicate the proper leveling off altitude for starting the cruise phase of the flight.
- B. When the mode for maximum range is selected the instrument should indicate:
  - 1. The best speed to be flown for maximum range.
  - 2. The best altitude to be flown for maximum range.
  - 3. The range remaining in nautical miles if the aircraft is flown in compliance with the indicated speed and altitude. This range is to be based on the fuel remaining less a fuel reserve which can be manually set by the pilot. Also, this range indication will be based on a minimum time descent path with an idle throttle setting at the end of the flight.
  - 4. The endurance remaining in minutes if the aircraft is flown according to the speed and altitude indicated by the instrument for maximum range mode. This endurance is to be based on same assumptions in (3) above.
  - 5. The time to start a maximum range descent with an idle throttle setting at end of flight. (This feature will permit obtaining a slightly longer range than the indicated value, which is based upon a minimum time descent.)

- C. When the mode for maximum endurance is selected the instrument should indicate:
1. The best speed to be flown for maximum endurance.
  2. The best altitude to be flown for maximum endurance.
  3. The endurance remaining in minutes if the aircraft is flown in accordance with the computed speed and altitude indicated. This endurance is to be based on the fuel remaining less a fuel reserve which can be set by the pilot. This endurance will also be based on a minimum time descent path with an idle throttle setting at end of flight.
  4. The range remaining in nautical miles if the aircraft is flown at the speed and altitude indicated by the instrument for best endurance. This range is to be based on same assumptions as in (3) above.
- D. When the mode for maximum range at constant altitude is selected the instrument should indicate:
1. The best speed to be flown at present pressure altitude in order to achieve maximum range at this altitude.
  2. The range remaining in nautical miles if aircraft is flown according to the speed indicated by the instrument and if the present pressure altitude is held constant; this range to be based on fuel remaining less the reserve quantity set by pilot, and a minimum time descent with an idle throttle setting at the end of the flight.

3. The endurance remaining in minutes, this value to be based on same assumptions as (2) above.
  4. The time to start a maximum range descent from the constant (cruising) pressure altitude in D. 2. (This feature will permit obtaining slightly longer range than the indicated value, which is based on a minimum time descent).
- E. When the mode for maximum endurance at constant altitude is selected the instrument should indicate:
1. The best speed to be flown at present pressure altitude in order to achieve maximum endurance at this altitude.
  2. The endurance remaining in minutes if aircraft is flown according to the speed indicated by the instrument and if present pressure altitude is held constant; this endurance to be based on fuel remaining less the reserve quantity set by the pilot and a minimum time descent at an idle throttle setting.
  3. The range remaining in nautical miles, this value to be based on same assumptions as (2) above.
- F. When the descent mode is selected the instrument should indicate:
1. The proper speed to be flown for a maximum range descent at an idle throttle setting.

### III Flight Performance And Parameter Limitations

To provide a margin of maneuverability for the aircraft the flight altitude indicated by the instrument should not exceed that at which the aircraft can achieve a 300 ft/min rate of climb at normal rated power. This is the cruise ceiling as defined by

military specifications.<sup>(1)</sup>

The limitations of the other parameters used in the instrument and displayed by the instrument should be those appropriate for the particular aircraft in question when operating from sea level to its absolute ceiling.

#### IV Manual Inputs

The instrument should have provision for the following manual inputs upon which the output of the instrument will be based in part:

##### A. Head Wind

There should be provision for the pilot to set in a value of head wind up to  $\pm 150$  knots. The control should be located so that the pilot has easy access to it during flight.

##### B. External Stores And Configuration Changes

Provision should be made for altering the outputs of the system to account for various external stores, combinations of these stores, and configuration changes, this to be done through the use of manual inputs. The manual input controls do not necessarily have to be easily accessible during flight, if some means for removing the appropriate inputs when the external stores (representing combat load) are expended is provided. This could be a manual control which might be called the "combat load jettison control," or could be an automatic control.

##### C. Internal Store And Weight Adjustment

Provision should be made for manually setting in weight adjustment compensation. The weight adjustment may be required because of difference in basic weight from one aircraft of the same model

to another, because of internal modifications to the aircraft or because of internal stores. This control need not necessarily be accessible to the pilot except for the readjustment necessary when the internal stores are expended. This readjustment might also be accomplished in flight by the combat load jettison control described in IV B

#### D. Fuel Reserve

Provision should be made for a control by which the pilot can set in the fuel reserve used in computing the values of range and endurance. This control should be easily accessible to the pilot.

#### E. Calibration

Some type of manual adjustment should be provided so that the instrument can be adjusted to read correctly after or during a flight test of the instrument aircraft combination. This adjustment is expected to be a fine adjustment to compensate for small errors in the system.

#### F. Fuel Quantity Setting

Means should be provided by which the fuel quantity aboard the aircraft before starting the engine may be set.

### V Accuracy

The accuracy of the system should be looked at from two view points

- A. The actual range and endurance that will be achieved by flying the indicated speeds and altitude compared to the range and endurance indicated by the instrument. These values should be:

1. Range - indicated range within 3% or 15 nautical miles of actual range, whichever is larger.
  2. Endurance - indicated endurance within 3% or 2 minutes of actual endurance, whichever is larger.
- B. The actual range and endurance achieved by flying according to the flight mode selected compared to the optimum range and endurance for the particular flight mode.
1. Range - actual range within  $1\frac{1}{2}\%$  or 7 nautical miles of optimum whichever is larger.
  2. Endurance - actual endurance within  $1\frac{1}{2}\%$  or 1 minute of optimum whichever is larger.

## VI Power Requirements

The system should be designed to be operated using the electrical power available in the aircraft and not over load the electrical system when it is operated simultaneously with other components of the aircraft requiring electrical power. In general power requirements of the instrument should be kept as low as possible.

## VII Size and Weight Requirements

No specific specifications can be set other than to outline the philosophy which should be followed.

Both size and weight should be held to a minimum by maximum employment of miniaturization and light weight parts. Control panel and display panel should be made as small as feasible and other components should be designed such that they can be remotely located.



## VIII Operating Requirements

### A. Temperature Requirements

The system should be able to withstand a temperature of  $-55^{\circ}\text{C}$  for four hours and temperature of  $+70^{\circ}\text{C}$  for four hours and still operate properly.

### B. Acceleration Requirements

The system as mounted in the aircraft should be designed to withstand without damage or impairment in operation an acceleration of 20 times that of gravity suddenly applied and one of seven times that of gravity applied for five minutes.

### C. Humidity Requirements

The system should be designed to operate indefinitely in atmospheres with 100% relative humidity.

### D. Pressure Requirements

The system should be capable of proper functioning at atmospheric pressures from - 1000 feet to the pressure at the absolute ceiling of the aircraft for which the system is designed.

## IX Display

The display presented to the pilot should be as small as possible and still be easy to read.

The display should be such that the pilot can tell at a glance which flight mode has been selected and whether or not he is flying at the speed and altitude indicated by the instrument for that particular flight mode.

The display should contain the computed values of range and endurance and as many of the manual inputs as feasible.

## CHAPTER 3

### INSTRUMENT DISPLAYS, CONTROLS, AND PRE-FLIGHT SETTINGS

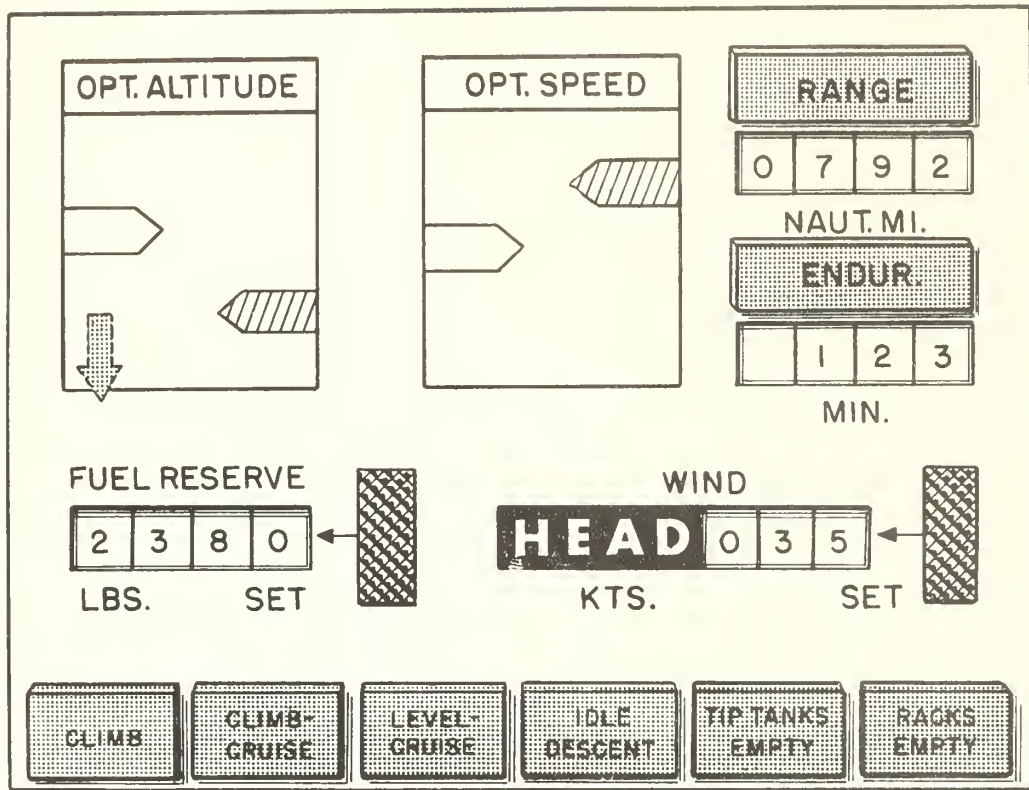
The purpose of this chapter is to formulate the general arrangement of the instrument display, controls and pre-flight settings. The philosophy of placing the essential informational display and in-flight controls on a single display and control panel in front of the pilot has been adopted, using the minimum space possible. All pre-flight adjustments, such as meteorological data on tropopause altitude, stratosphere (isothermal) temperature, unmetered fuel, initial configuration, and basic weight adjustment, are relegated to a pre-flight adjustment panel which may be placed in a less convenient (and less critical) location away from the display and control panel.

#### A. Display and Control Panel

The display and control panel contains the main informational displays and manual controls. Its intended location is near the main flight instruments such as artificial horizon, altimeter, airspeed, etc. It is so designed to make maximum utilization of space and still provide a large quantity of information. The proposed method of display is outlined below and shown in Figure 3-1.

##### 1. Altitude Deviation Indicator

This type of presentation provides the essential information on altitude control to the pilot, without the necessity of covering an entire range of altitudes from sea level to maximum ceiling



3-1 Display-control panel

which would require more space. (Actually, as will be developed in succeeding chapters, it is a  $\frac{W}{\delta}$  deviation indicator.) It informs the pilot whether he is above or below the optimum altitude for the selected mode, and the approximate magnitude (within the range of the instrument) of the deviation. It is felt that approximately  $\pm 4000$  ft of altitude should be the range of the instrument, at the cruise altitude. The left-hand needle is the optimum reference and is fixed at the center of the scale. The right-hand needle moves up and down as the actual altitude becomes higher than, or lower than optimum, respectively. Limit stops prevent the right needle from moving out of view, so that the proper sense of gross deviations will always be evident.

## 2. Airspeed Deviation Indicator

The same type of two needle display is used for airspeed deviations from optimum, as for altitude deviations. A desirable range is approximately  $\pm 100$  knots TAS. Actually, Mach number is a more convenient parameter to employ within the computer and the indicator could be more simply instrumented as a Mach number deviation indicator. The desirable range of the instrument as a Mach number deviation indicator is  $\pm 0.15$  M.

## 3 Range and Endurance Mode Selection Switches

Two push-button type switches labelled "RANGE" and "ENDURANCE" respectively are provided in the upper right-hand side of the display-control

panel. The switches are back-lighted when in the "IN" position. The two switches are mutually exclusive, so that only one may be selected at a time. The inscriptions are legible both with and without back-lighting. The "Range and Endurance" switches perform three functions.

- a. They permit mode selection: i. e., pushing the "RANGE" switch in, selects the maximum range mode within the computer.
- b. They indicate the mode selected when pushed in by back-lighting.
- c. They are placed immediately above two windows in which counter-type indicators present the predicted range and endurance, and act as labels for these indicators.

#### 4. Flight Path Selection Switches

There are four switches in this group: "CLIMB", "CLIMB-CRUISE", "LEVEL-CRUISE", and "DESCEND". These switches are also back-lighted and mutually exclusive. The pilot selects the appropriate switch for the type of flight path he wishes to fly.

The function of each Flight Path Selection Switch will be described. Depressing the "CLIMB" switch causes the computer to indicate the best climbing airspeed schedule for the particular mode selected, (as a deviation from actual airspeed), on the airspeed deviation indicator. At the same time, it

causes the altitude deviation indicator to show the deviation from the optimum cruising altitude (which will vary with weight, configuration, temperature, etc.). Also it causes the indicated range and endurance to be computed on the basis of cruising at the optimum conditions for the mode (Range or Endurance) selected. Most of the climb to cruising altitude would be accomplished with the altitude deviation indicator resting against the lower stop. In the latter portion of the climb, the needle will move into coincidence with the reference as the best cruising altitude is reached. At this time the pilot may push either the "CLIMB-CRUISE", or the "LEVEL-CRUISE" switch. Depending on which is chosen, the proper range, endurance, altitude deviation and airspeed deviation are indicated for the mode (Range or Endurance) selected, and as influenced by other factors such as weight, configuration, temperature, etc. The "CLIMB-CRUISE" position results in maximum range or endurance.

The "LEVEL-CRUISE" position results in a slightly lower range than obtainable from using a climb-cruise flight path. The optimum cruising air-speed for the particular mode selected is indicated as a deviation of actual air-speed from optimum, and will vary as the weight of the aircraft changes. In order to simplify the instrumentation, the standard altimeter will be used to maintain the altitude, with the altitude deviation indicator showing the deviation of the actual (level-cruise) altitude from the optimum climb-cruise altitude. This will enable the pilot to select some even altitude, generally near the



optimum, if he so desires.

At a point on the cruise path a small downward-pointing arrow on the altitude deviation indicator will light up, indicating the time to initiate a maximum range idle rpm descent to arrive at sea level with the preset value of fuel reserve. If the pilot elects to use this type of descent, he may at this time push the "DESCEND" switch. This switch causes the computer to indicate the proper idle rpm descent airspeed schedule. The altitude deviation indicator loses its usefulness in this phase, and should be disregarded by the pilot. It may be desirable to blank off this indicator for the descent phase, either by raising a flag, dropping a screen over the face of the instrument, or possibly having needles that withdraw from sight by solenoid action.

#### 5. Fuel Reserve Set Knob

This knob permits selecting a given weight of fuel, as a reserve. Fuel reserve is defined as the weight of fuel remaining at the end point of a minimum time (operational type) descent. Any adjustment of this fuel reserve will naturally affect the computed values of predicted range and endurance, as described in Chapter 7.

#### 6. "TIP TANKS EMPTY" Switch

In order to explain the reason for including this switch, it is necessary to investigate the problem of fuel quantity measurement. In most tactical jet aircraft the fuel quantity

indicator measures only the quantity of fuel in the fuselage, or main fuel cells and all fuel to the engine feeds from the main fuel cells. Fuel contained in tip or wing auxiliary tanks is not directly measurable. In order to obtain a reasonably accurate, continuous value of fuel remaining it is proposed to separate the problem of determining the fuel remaining into two portions:

a. Tip And/Or Wing Tank's Containing Fuel

The fuel aboard is determined by subtracting the integrated output of a fuel flow meter, or mass flow meter, located at the inlet to the main cell, from the sum of the main fuel cell quantity indication (provided this can be measured accurately) plus the pre-flight setting of unmetered fuel.

b. Tip and Wing Tanks empty

In this case, the fuel quantity would be obtained by direct measurement of fuel contained in the main cell provided means can be provided for accurately measuring the fuel quantity in this cell.

The switchover from (a) and (b) would be manually initiated when tip and wing tanks are emptied by the pilot pushing in the "TIP TANKS EMPTY" switch. The reason for avoiding an automatic changeover is the possibility of various types of errors being introduced. For example, a fuel quantity switch set at, for example, 80% of the main



fuel cell capacity might be used for switch-over. However, in case the tip or wing tanks had failed to transfer, (not an infrequent occurrence) there would be a gross weight error introduced equal to the amount of fuel remaining in the auxiliary tanks. Another possibility would be to connect the switchover directly to the fuel transfer switch. This is not too satisfactory, since any time the transfer switch was off, and there was fuel remaining in the auxiliary tanks, there would again be a gross weight error. One instance in which this situation would arise is when a pilot (by personal choice or squadron doctrine) does not begin transferring fuel until after take-off and a portion of the climb has been completed.

Since the pilot normally knows when the auxiliary tanks are actually empty the manual switch was adopted. The "TIP TANKS EMPTY" is also back-lighted when it is in the "in" position.

#### 7. "RACKS EMPTY" Switch

The reason for adding this switch is to obviate the necessity of the pilot's using the pre-flight adjustment panel after expending a combat load. This switch would automatically readjust the configuration setting to the "CLEAN" position, or to another appropriate configuration which would be more or less fixed in nature.

## 8. "WIND SET" Adjustment

The "WIND SET" adjustment is for the purpose of entering the value of head or tail-wind encountered in flight. The knurled knob adjustment allows the wind velocity counter to be rotated in order to indicate the wind velocity (in knots) along the flight path. Also, as the counter passes through the zero position the indication will change from "HEAD" to "TAIL".

## B. Pre-Flight Adjustment Panel

This panel, shown in Figure 3-2, is intended to be accessible to the pilot in flight, although it need not be mounted in any of the critical forward areas of the cockpit. Normally, the pilot need not make any adjustments on this panel during flight. The various settings in this panel are:

### 1. "CONFIGURATION SET"

This switch is of primary importance, as it influences the flight parameters such as range and endurance to a great extent. This switch is intended to account for additional items mounted in or on the aircraft, from both a weight and a drag standpoint. Naturally, internal stores will add weight, but the drag increment will be zero. The knurled set knob is rotated until the correct configuration at time of take off appears in the window. An internal stop is located at a selected configuration, so that the configuration drum will return to this setting when the "RACKS EMPTY" switch on the display and control panel is pushed in.

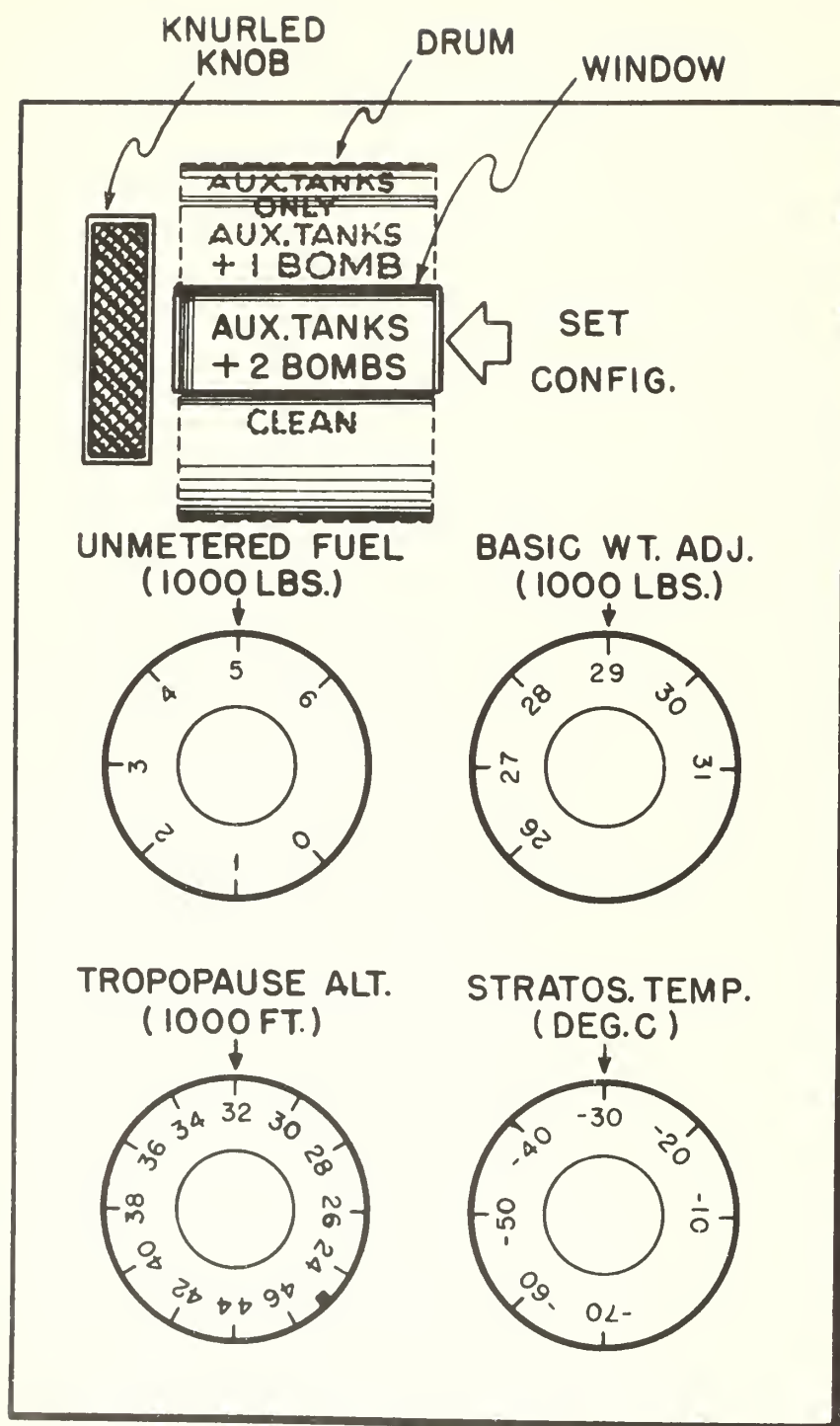


Fig 3-2 Pre-flight adjustment panel

## 2. "TROPOPAUSE ALTITUDE"

This knob is set to the altitude at which the tropopause occurs, based on the best current meteorological data.

## 3. "STRATOSPHERE TEMPERATURE"

This knob is set to the temperature of the stratosphere (isothermal layer) based on the best current meteorological data.

## 4. "BASIC WEIGHT ADJUSTMENT"

This knob is adjusted to give the correct basic weight of the clean aircraft, omitting any of the additional stores accounted for by the "CONFIGURATION SET" knob and also omitting the total fuel load.

## 5. "UNMETERED FUEL WEIGHT"

This knob is used to set the weight of unmetered fuel carried in auxiliary tanks.



## CHAPTER 4

### CONDITIONS FOR ALL-OUT MAXIMUM RANGE IN CRUISING FLIGHT

It can be demonstrated analytically for specific turbojet aircraft with either Mach-limited or thrust-limited cruise characteristics that all out maximum range in cruising flight may be obtained by flying at a constant value of  $\frac{W}{\delta}$  and constant Mach number.

For aircraft with Mach-limited cruise characteristics the concept of constant  $\frac{W}{\delta}$ , constant Mach number cruising is developed in Derivation Summaries 3, 4 and 7 of "Cruise Control Techniques for Turbojet Aircraft" WADC Technical Report 55-246.<sup>(2)</sup> This development is reproduced on the following pages.

For turbojet aircraft with thrust-limited cruise characteristics the curve of specific range vs  $\frac{W}{\delta}$  is similar to the curve for aircraft with Mach-limited cruise characteristics except that in the case of the thrust-limited aircraft flight performance, limitations are reached at values of  $\frac{W}{\delta}$  lower than that at which the specific range reaches its theoretical maximum value. The maximum range is therefore obtained by flying at the maximum value of  $\frac{W}{\delta}$  permitted by the flight performance limitation. The flight performance limitation is the cruise ceiling which has been defined in Military Specification, MIL-C-5011A, dated 5 Nov. 1951, as that altitude at which a 300 ft/ min rate of climb can be achieved at normal rated thrust. The cruise ceiling increases as the aircraft weight decreases (as a result of fuel expenditure).

As shown below, it is seen that at the cruise ceiling the value of  $\frac{W}{\delta}$  is a constant.

### Derivation Summary 3. Useful lift and drag relations.

Assuming approximately steady-state level flight:

$$W = L = C_L \frac{\rho}{2} S V_T^2 = C_L \left( \frac{P}{2 R_q T} \right) M^2 S (k R_q T) \quad (1)$$

$$= C_L K_s \rho M^2 \quad (2)$$

where  $K_s = \frac{kS}{2}$

Another useful form is:

$$\frac{W}{\delta} = (K_s \rho_{sl}) C_L M^2 \quad (3)$$

Similarly for the drag:

$$D = C_D \frac{\rho}{2} S V_T^2 \quad ; \quad D = C_D K_s \rho M^2 \quad (4), (5)$$

$$\frac{D}{\delta} = (K_s \rho_{sl}) C_D M^2 \quad (6)$$

The basic incompressible form of the drag coefficient is:

$$C_D = C_{D_o} + C_{D_i} \quad (7)$$

$$= C_{D_o} + \frac{C_L^2}{\pi e (AR)} \quad (8)$$

where  $C_{D_o}$  and  $e$  can be considered constants below the drag rise.

The expression for maximum  $L/D$  is obtained as follows:

$$\frac{C_D}{C_L} = \frac{C_{D_o}}{C_L} + \frac{C_L}{\pi e (AR)}$$

$$\frac{d(C_D/C_L)}{dC_L} = 0 = -\frac{C_{D_o}}{C_L^2} + \frac{1}{\pi e (AR)}$$

Therefore at  $(L/D)_{\max}$

$$C_{D_i} = \frac{C_L^2}{\pi e (AR)} = C_{D_c} \quad (9)$$

and

$$C_D = 2C_{D_o} \quad (10)$$

Therefore

$$\left( \frac{L}{D} \right)_{\max} = \frac{\sqrt{C_{D_o} \pi e (AR)}}{2C_{D_o}} = \sqrt{\frac{\pi e (AR)}{4C_{D_o}}} = \text{const} \quad (11)$$

The expression for maximum  $(C_L^{1/2}/C_D)$  is obtained as follows:

$$\frac{C_D}{C_L^{1/2}} = \frac{C_{D_o}}{C_L^{1/2}} + \frac{C_L^{3/2}}{\pi e (AR)}$$

$$\frac{d\left(\frac{C_D}{C_L^{1/2}}\right)}{dC_L} = 0 = -\frac{1}{2} \frac{C_{D_o}}{C_L^{3/2}} + \frac{3}{2} \frac{C_L^{1/2}}{\pi e (AR)}$$

Therefore at  $(C_L^{1/2}/C_D)_{\max}$ :

$$C_{D_i} = \frac{C_L^2}{\pi e (AR)} = \frac{1}{3} C_{D_c} \quad (12)$$

and

$$C_D = \frac{4}{3} C_{D_o} \quad (13)$$

Therefore:

$$\left( \frac{C_L^{1/2}}{C_D} \right)_{\max} = \frac{\sqrt{\frac{4}{3} C_{D_o} \pi e (AR)}}{\frac{4}{3} C_{D_o}} = \text{const} \quad (14)$$



#### Derivation Summary 4. Cruise relations for turbojet powered aircraft.

Assumptions:

- Steady-state, level-flight cruise at constant altitude.
- No-drag-rise effects ( $C_{D_0}$  and  $e$  constant).
- Constant thrust specific fuel consumption,  $c_f$ .
- Still air.

The basic expression for specific range is

$$(SR) = \frac{V_T}{w_f} = \frac{V_T}{c_f D} \quad (1)$$

Substituting from Eqs. (1) and (4) of Derivation Summary 3

$$(SR) = \frac{\sqrt{\frac{W}{C_L \frac{\rho}{2} S}}}{c_f C_D \frac{\rho}{2} S \left( \frac{W}{C_L \frac{\rho}{2} S} \right)} = \frac{1}{c_f} \sqrt{\frac{2}{\rho S W}} \times \left( \frac{C_L^{1/2}}{C_D} \right) \quad (2)$$

For maximum specific range at constant altitude and a given weight

$$(SR)_o = \frac{1}{c_f} \sqrt{\frac{2}{\rho S W}} \left( \frac{C_L^{1/2}}{C_D} \right)_{\max} \quad (3)$$

where the subscript "o" will be used throughout this study to refer to the  $(C_L^{1/2}/C_D)_{\max}$  condition. Assuming incompressible flow (valid for flight below the drag rise), the drag coefficient can be represented by:

$$C_D = C_{D_0} + \frac{C_L^2}{\pi e (AR)} \quad (4)$$

Using this, the expression for  $(C_L^{1/2}/C_D)_{\max}$  obtained in Derivation Summary 3 is

$$\left( \frac{C_L^{1/2}}{C_D} \right)_{\max} = \frac{\sqrt[4]{\frac{1}{3} C_{D_0} \pi e (AR)}}{\frac{4}{3} C_{D_0}} = \text{const} \quad (5)$$

It can be seen from Eq. (5) that  $(C_L^{1/2}/C_D)_{\max}$  is essentially constant for all altitudes and weights below the drag rise.

The maximum specific range is then simply

$$(SR)_o = \text{const} \times \sqrt{\frac{1}{W \sigma}} \quad (6)$$

The corresponding optimum cruise speed at  $(C_L^{1/2}/C_D)_{\max}$  is:

$$V_{T_o} = \sqrt[4]{\frac{3 W^2}{C_{D_0} \pi e (AR) \left( \frac{\rho}{2} S \right)^2}} ; \quad V_{T_o} = \text{const} \times \sqrt{\frac{W}{\sigma}} \quad (7),(8)$$

In terms of Mach number:

$$M_o = \sqrt[4]{\frac{3 W^2}{C_{D_0} \pi e (AR) K_s p^2}} ; \quad M_o = \text{const} \times \sqrt{\frac{W}{\delta}} \quad (9),(10)$$

From Eq. (6), Derivation Summary 2

$$M_{(L/D)\max} = \sqrt[4]{\frac{W^2}{C_{D_0} \pi e (AR) K_s p^2}}$$

Therefore

$$(M_o/M_{(L/D)\max}) = \sqrt[4]{3} = 1.316 = \frac{1}{0.760} \quad (11)$$

The maximum range for cruise at constant altitude is:

$$\begin{aligned} R_{\max} &= \int_{W_1}^{W_2} (SR)_o dW = \int_{W_1}^{W_2} \left( \frac{C_L^{1/2}}{C_D} \right)_{\max} \frac{1}{c_f} \sqrt{\frac{2}{\rho S}} \frac{dW}{\sqrt{W}} \\ &= \frac{26.0}{c_f} \frac{\sqrt[4]{C_{D_0} e (AR)}}{C_{D_0}} \sqrt{\frac{W_1}{\sigma S}} \left[ 1 - \left( \frac{W_2}{W_1} \right)^{1/2} \right] \text{ naut mi} \end{aligned} \quad (12)$$

This is similar to the Breguet Range Equation modified for constant altitude cruise of turbojet powered aircraft.



### Derivation Summary 7. Origin of constant $W/\delta$ concept for Mach-limited cruise.

Assumptions:

- Approximately steady-state, level-flight cruise.
- Cruise at constant Mach number equal to  $M_c$ .
- No important drag-rise effects for cruise at  $M_c$  ( $C_{D_0}$  and  $e$  still essentially constant).
- Constant thrust specific fuel consumption,  $c_f$ .
- Still air.

The basic expression for specific range is

$$(SR) = \frac{V_T}{w_f} = \frac{V_s M}{c_f D} \quad (1)$$

For cruise above 35,000 ft, the speed of sound can be assumed to be constant. If the cruise Mach number is constant at the critical Mach number, the specific range becomes

$$\begin{aligned} (SR) &= \frac{V_s M_c}{c_f} \times \frac{1}{D} \\ &= \text{const} \times \left(\frac{L}{D}\right) \times \frac{1}{W} \end{aligned} \quad (2)$$

Therefore, for maximum specific range the optimum altitude is that for minimum drag, which is equivalent to cruise at the altitude for maximum  $L/D$ . Up to the critical Mach number, the drag coefficient can be assumed to be

$$C_D = C_{D_0} + \frac{C_L^2}{\pi e (AR)}$$

Using this, it is shown in Derivation Summary 3, Eq. (11) that

$$\begin{aligned} \left(\frac{L}{D}\right)_{\max} &= \sqrt{\frac{\pi e (AR)}{4 C_{D_0}}} \\ &= \text{const} \end{aligned} \quad (3)$$

To determine the variation of the optimum cruise altitude with aircraft weight for Mach-limited cruise it is necessary to go back to the original expression for  $L/D$ :

$$\frac{L}{D} = \frac{W}{C_D K_s \rho M_c^2}$$

Therefore

$$\begin{aligned} \left(\frac{L}{D}\right)_{\max} &= \frac{W}{2 C_{D_0} K_s \rho_{(L/D)_{\max}} M_c^2} \\ &= \text{const} \times \frac{W}{\delta_{(L/D)_{\max}}} \end{aligned} \quad (4)$$

However, since  $(L/D)_{\max}$  is a constant value as can be seen from Eq. (3), then Mach-limited cruise occurs at a constant value of  $W/\delta$ . This value is given by

$$\left(\frac{W}{\delta}\right)_{\text{opt}} = \frac{W}{\delta_{(L/D)_{\max}}} = \text{const} \quad (5)$$

However, from Eq. (3) of Derivation Summary 3

$$\frac{W}{\delta} = (K_s \rho_{sl}) C_L M^2 \quad (6)$$

Substituting the value of  $C_L$  for  $(L/D)_{\max}$  which can be obtained from Eq. (9) of Derivation Summary 3, the optimum  $W/\delta$  as given by this simple theory is:

$$\left(\frac{W}{\delta}\right)_{\text{opt}} = (K_s \rho_{sl}) M_c^2 \sqrt{C_{D_0} \pi e (AR)} \quad (7)$$

The fundamental relationship for rate of climb is: (3)

$$\frac{R}{C} = \frac{(F - D) V}{W} \text{ which may be written:}$$

$$\frac{R}{C} = \frac{\left(\frac{F}{\delta} - \frac{D}{\delta}\right)V}{\frac{W}{\delta}} \quad (4-1)$$

Considering that the universal gas-law holds it can be written that:

$$\begin{aligned} D &= \frac{C_D}{2} \rho V^2 S = \text{Constant} \times \rho V^2 \\ &= \text{Constant} \times \frac{p}{RT} V^2 \\ &= \text{Constant} \times \frac{p_{SL_o}}{p_{SL_o}} \frac{T_{SL_o}}{T_{SL_o}} \frac{p}{RT} V^2 \\ &= \text{Constant} \times \frac{\delta}{\theta} V^2 \end{aligned} \quad (4-2)$$

The rate of climb can then be written

$$\frac{R}{C} = \frac{\left(\frac{F}{\delta}\right)V - \text{Constant} \frac{V^3}{\theta}}{\frac{W}{\delta}} \quad (4-3)$$

The power setting is the normal rated thrust condition. Therefore  $\frac{F}{\delta} \approx \text{Constant}$  when it is observed that  $\frac{F}{\delta}$  for a given power setting only changes slightly with velocity changes in the operating range of velocities.

$$\text{Thus: } \frac{R}{C} = \frac{C_1 V - C_2 \frac{V^3}{\theta}}{\frac{W}{\delta}} \quad (4-4)$$

Holding  $\frac{W}{\delta}$  constant and solving for the value of  $V$  that gives  $[\frac{R}{C}]_{\max}$  by differentiating the equation for  $\frac{R}{C}$  and setting the derivative equal to zero, :

$$C_1 - \frac{3 V^2 C_2}{\theta} = 0 \quad \left\{ \text{condition for } [\frac{R}{C}]_{\max} \right\}$$

or 
$$V = \left[ \frac{C_1 \theta}{3 C_2} \right]^{\frac{1}{2}} \quad (4-5)$$

substituting this value into the equation for  $\frac{R}{C}$  gives:

$$[\frac{R}{C}]_{\max} = \frac{C_1^{\frac{3}{2}} \left[ \frac{\theta}{3 C_2} \right]^{\frac{1}{2}} - \frac{1}{\sqrt{C_2}} \left[ \frac{C_1 \theta}{3} \right]^{\frac{3}{2}}}{\frac{W}{\delta}} \quad (4-6)$$

thus if  $[\frac{R}{C}]_{\max} = 300 \text{ ft/min} = \text{Constant}$  at the cruise ceiling, then  $\frac{W}{\delta}$  must also be constant at the cruise ceiling for any given value of  $\theta$ .

This fact can be verified for a given turbojet aircraft by plotting the 300 ft/min climb line on the normal thrust climb performance plot of  $\frac{R}{C}$  vs altitude for different weights, and then checking the value of  $\frac{W}{\delta}$  along this line. It will be noted that normally the cruise ceiling lies at altitudes in the isothermal layer, thus  $\theta = \text{Constant}$ .

Now the values of  $(\frac{W}{\delta})_R$  and  $M_R$  to be flown by the turbojet aircraft with thrust limited cruise characteristics to achieve all out maximum range must be derived.

First a plot of  $(\frac{W}{\delta})_R$  vs  $\theta$  at the cruise ceiling is made for any given configuration. This is accomplished by first plotting the cruise ceiling for a given configuration as a curve on a weight vs altitude plot. This is done for standard, hot, and cold days.

Using these curves, plots of  $(\frac{W}{\delta})_R$  vs  $\theta$  can be constructed as shown in Figure 4-1. Examination of the plots of  $(\frac{W}{\delta})_R$  vs  $\theta$  at the cruise ceiling indicates that they can be approximated by an expression of the form:

$$(\frac{W}{\delta}) \text{ at cruise ceiling} \cong (\frac{W}{\delta})_R \approx \text{Constant}_1 + \text{Constant}_2 \theta \quad (4-7)$$

Next the optimum speed for maximum range,  $M_R$ , must be determined as a function of  $\frac{W}{\delta}$ . This is accomplished by taking the curves of specific range vs Mach number for different weights, altitudes, and wind velocities; and then making a cross plot of  $\frac{W}{\delta}$  vs  $M_R$ , where  $M_R$  is taken as the velocity at which specific range is a maximum. The result will be a set of curves for each configuration with wind as a parameter as shown in Figure 4-2. Plotting these curves for standard, hot, and cold days indicates that ambient temperature effects are negligible. Thus a single set of curves as shown in Figure 4-2 can be employed to determine  $M_R$  as a function of  $\frac{W}{\delta}$  for a given configuration.

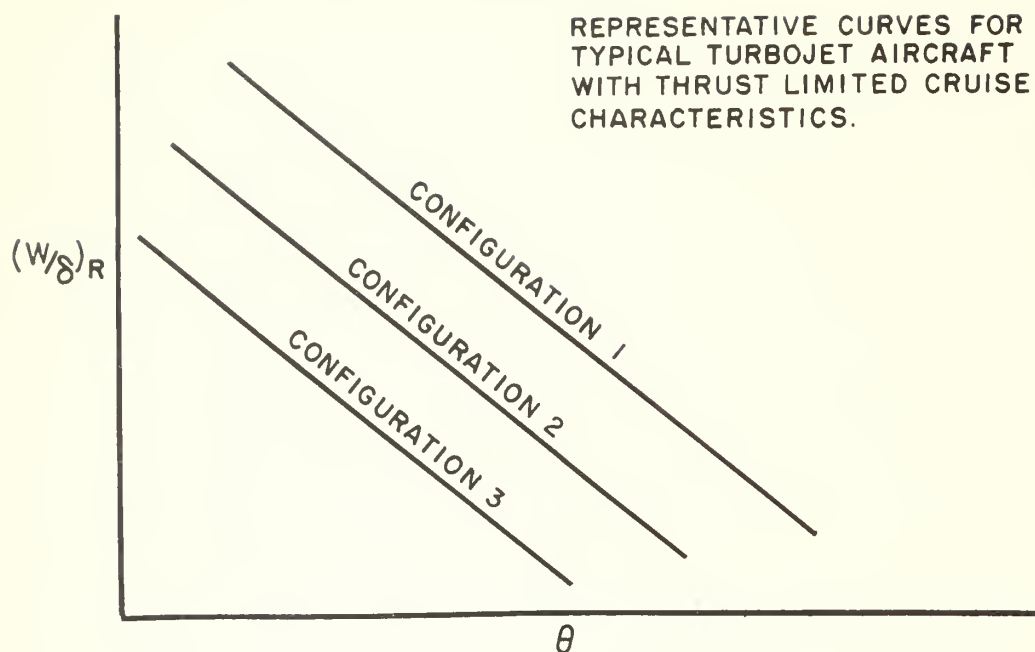


Fig. 4-1 Variation of  $(W/\delta)_R$  with temperature ratio,  $\theta$ .

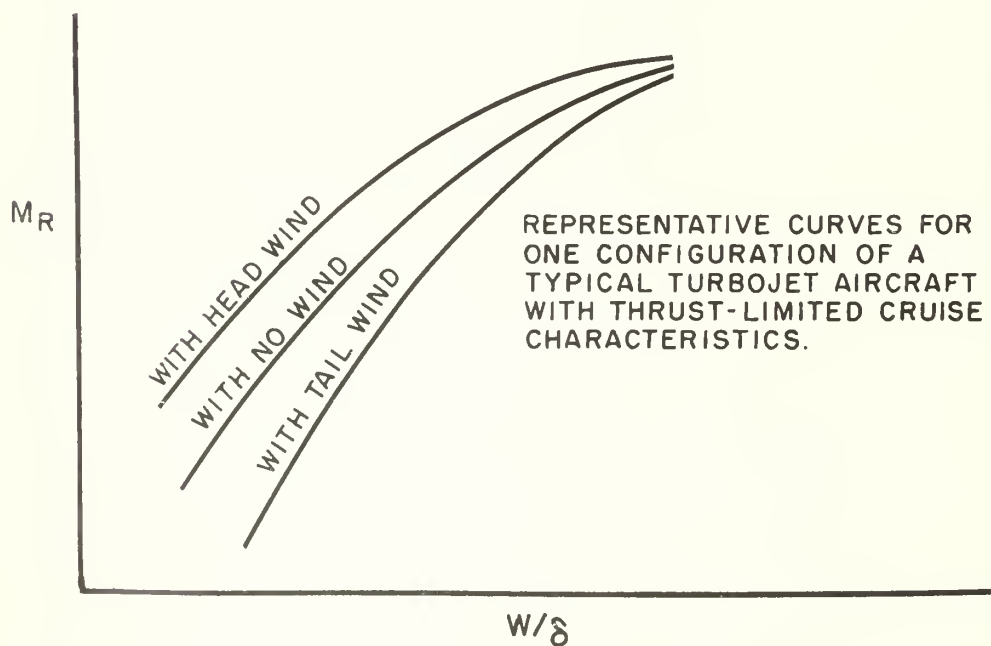


Fig. 4-2 Optimum range Mach number,  $M_R$ , variation with  $W/\delta$ .

## CHAPTER 5

### INSTRUMENTATION FOR OBTAINING THE VALUE OF $\frac{W}{\delta}$ FOR MAXIMUM RANGE

The method used to determine the optimum value of  $\frac{W}{\delta}$  for maximum range,  $(\frac{W}{\delta})_R$ , for an aircraft with thrust-limited cruise characteristics has been outlined in Chapter 4. Examination of Figure 4-1 indicates that in order to obtain the value of  $(\frac{W}{\delta})_R$  it is necessary to know:

- a. the aircraft configuration, and
- b. the temperature ratio existing at the cruise altitude.

In order to predict  $(\frac{W}{\delta})_R$  some sort of stored atmospheric temperature data is necessary. In Appendix A, the concept of storing this data by means of the tropopause altitude and isothermal temperature is developed.

It will be the purpose of this Chapter to demonstrate the manner of instrumenting the curve of  $(\frac{W}{\delta})_R$  vs  $\theta$  (Figure 4-1) and the curve of  $(\frac{W_1}{\delta_{atm}})$  vs  $\theta$  (Figure 5-1) and then, for any configuration solving for the intersection of these curves in order to predict  $(\frac{W}{\delta})_R$ . Figure 5-2 shows typical intersections of such curves.

The curve of  $(\frac{W_1}{\delta_{atm}})$  vs  $\theta$  is obtained by modifying a stored curve of  $\delta_{atm}$  vs  $\theta$ , by the quantity  $W_1$ . Two methods of storing the  $\delta_{atm}$  vs  $\theta$  curves were investigated. The first involved calculating the NACA standard day  $\delta_{atm}$  vs  $\theta$  curve and then

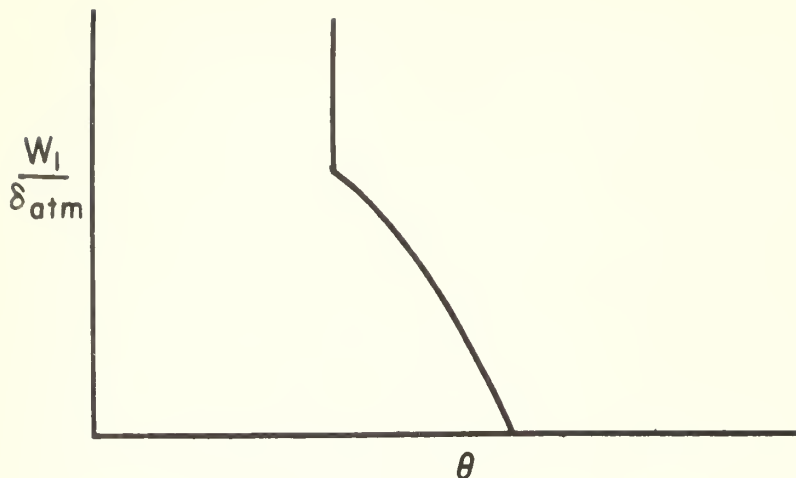


Fig. 5-1 Variation of  $(\frac{W_I}{\delta_{atm}})$  with  $\theta$

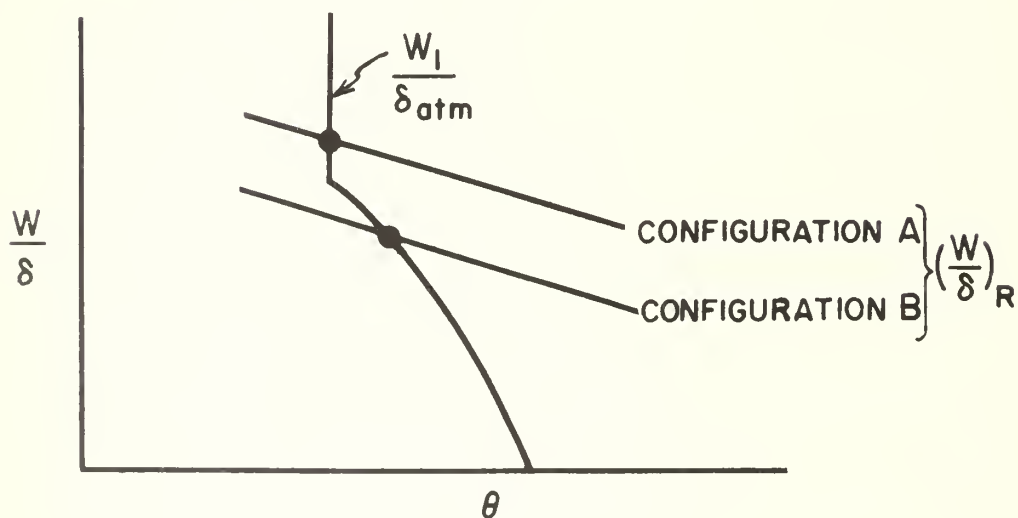


Fig. 5-2 Method of determining  $\theta$  using the intersection of  $(\frac{W}{\delta})_R$  and  $(\frac{W_I}{\delta_{atm}})$  curves, showing two typical intersections.



simply shifting this curve vertically and horizontally for non-standard days so that the tropopause break point was in the proper location. Unfortunately, the non-standard curves thus derived can diverge too far from the true curves for non-standard conditions due to the possible wide variations of tropopause altitude and isothermal temperature from standard. The second and most promising method involved instrumenting the formula of  $\delta_{\text{atm}}$  as a function of  $\theta$  directly.

The instrumentation of equation (A-19) might be accomplished using logarithmic potentiometers as shown in Figure 5-3. Considering  $\theta$  as the input, the proposed method of instrumenting equation (A-19) would yield the desired results for  $\delta_{\text{atm}}$  and  $\theta$  below the tropopause. Above the tropopause however, the input  $\theta$  value must be limited, and equal to  $\theta_B$  for all values of  $\delta$ . Since  $\theta$  represents a shaft angle input (driven by a servo-motor) to a potentiometer, it is necessary to prevent the servo-motor from driving in such a direction as to decrease  $\theta$  to a value smaller than  $\theta_B$ . A means of accomplishing this is shown in Figures 5-4 and 5-5. The mechanical switch operates from a mechanical differential gear connected to the  $\theta$  and the  $\theta_B$  shafts. When  $\theta - \theta_B$  is positive the switch is closed (allowing a  $\theta$  correction signal of either sign to reach the servo-motor). When  $\theta - \theta_B$  is zero or negative, this switch is open, and the  $\theta$  correction signal must pass through the phase-sensitive network to reach the servo-motor. The network is so placed as to allow only an "increase  $\theta$ " signal to pass, to the motor. In this manner the minimum value of  $\theta$  is effectively limited to  $\theta_B$ , but  $\theta$  can increase as necessary.

The curves of  $(\frac{W}{\delta})_R$  vs  $\theta$  must now be considered. Since rather high values of  $(\frac{W}{\delta})_R$  are the only ones of interest for long range predictions, an approximation to the  $(\frac{W}{\delta})_R$  versus  $\theta$  curves for various configurations might be accomplished by the method shown graphically in Figure 5-6. The sensitivity,  $S[\theta; \frac{W}{\delta}]$ , could easily be adjusted by a multiplicative factor to



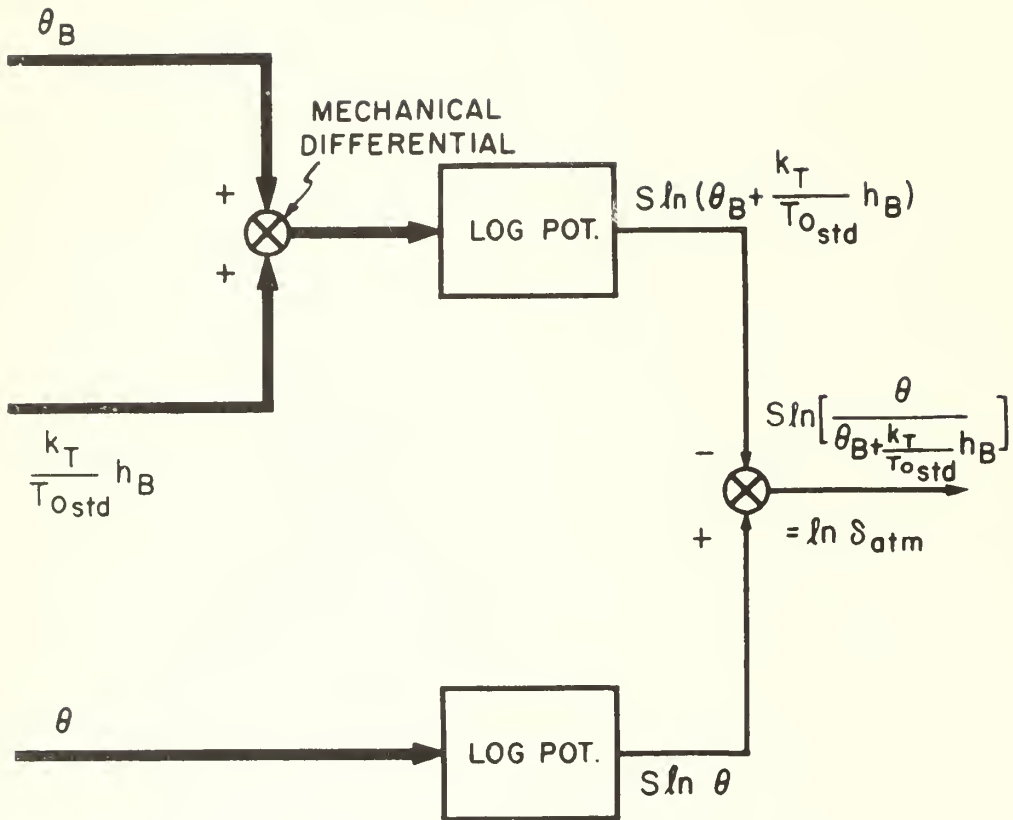


Fig. 5-3 Method of instrumenting  $\delta_{atm}$  as a function of  $\theta$ , using logarithmic potentiometers.

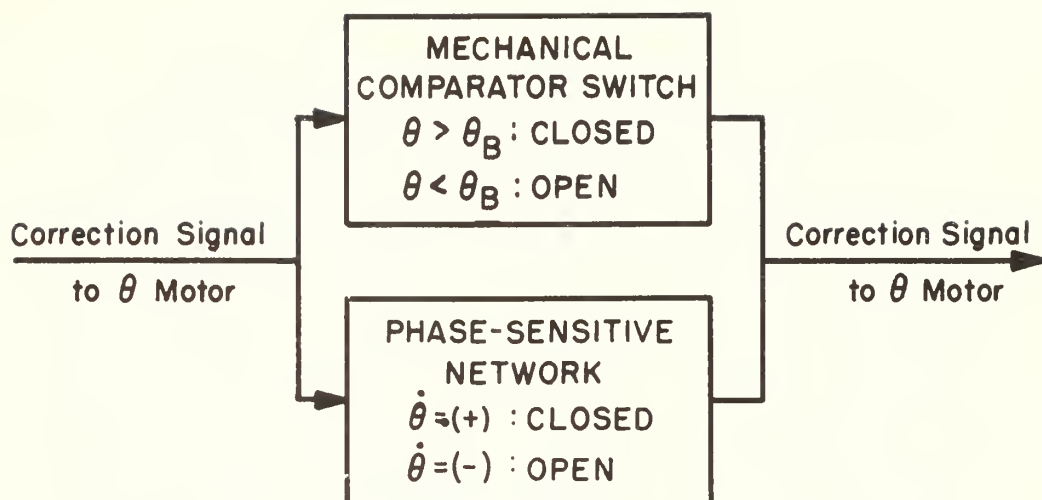


Fig. 5-4 Block diagram of  $\theta$ -limiting device.

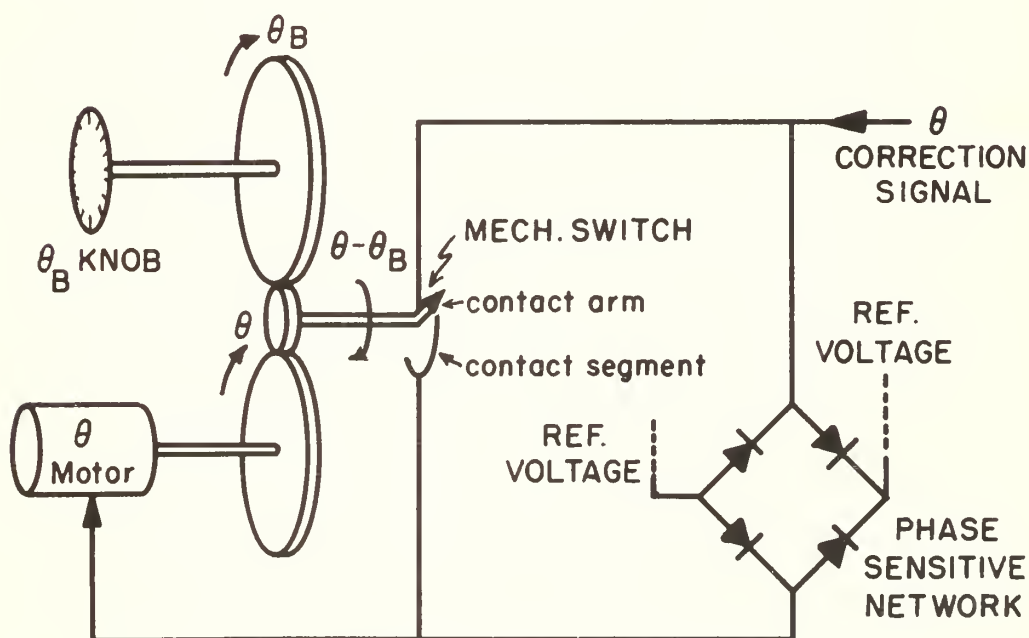


Fig. 5-5 Physical method of achieving a  $\theta$  limiting device.

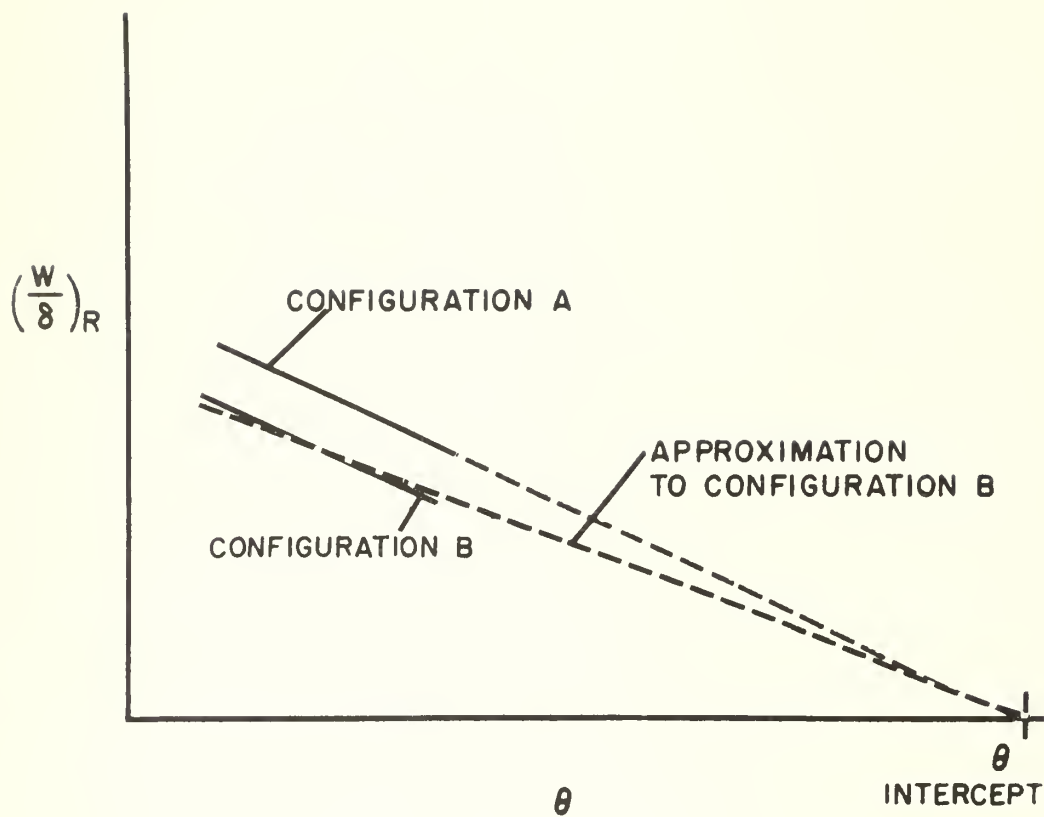


Fig. 5-6 Method of approximating  $(\frac{W}{\delta})_R$  curve for various configurations, by adjusting the slope of a line passing through  $\theta_{\text{intercept}}$ .

account for changes of configuration. The value of ( $\theta$  intercept) is held constant at the intersection of a  $(\frac{W}{\delta})_R$  curve for an intermediate configuration with the horizontal axis. Analysis of a representative case shows that this method of approximation results in a maximum error in  $(\frac{W}{\delta})_R$  of  $1\frac{1}{2}\%$  over the useful range of the curve. Further analysis showed that the above error amounts to less than 1% in range. The reason for this is that the specific range parameter increases as  $\frac{W}{\delta}$  decreases. In view of the above facts, it is felt that the proposed simplification is warranted.

Using the relation,

$$(\frac{W}{\delta})_R = S[\theta; \frac{W}{\delta}] (\theta - \theta_{\text{intercept}}) \quad (5-1)$$

the following can be written:

$$\ln (\frac{W}{\delta})_R = \ln S[\theta; \frac{W}{\delta}] + \ln (\theta - \theta_{\text{intercept}}) \quad (5-2)$$

Now, equation (5-2) may be instrumented by the method shown in Figure 5-7, where  $S[\theta; \frac{W}{\delta}]$  will be adjusted by the "CONFIGURATION SET" knob of the pre-flight adjustment panel

The overall system is shown in schematic form in Figure 5-8, where  $\ln (\frac{W}{\delta})_R$  is the output. The convenience of this logarithmic form for range computations lies in the fact that the specific range parameter  $\frac{V\delta}{w_f}$  must be multiplied by  $\frac{W}{\delta}$ , and this is easily accomplished by adding their logarithms.

In Figure 5-8, the temperature input in the climb-cruise and level cruise phase is led in from a direct temperature measuring source, and the stored atmospheric data is bypassed. An analysis of the effect of temperature on the computed value of range showed that for a typical turbojet aircraft with thrust-limited cruise characteristics, a 1% change in range resulted from a 6-8 degree Centigrade error in temperature input. It is probable that the error in meteorological data would not exceed

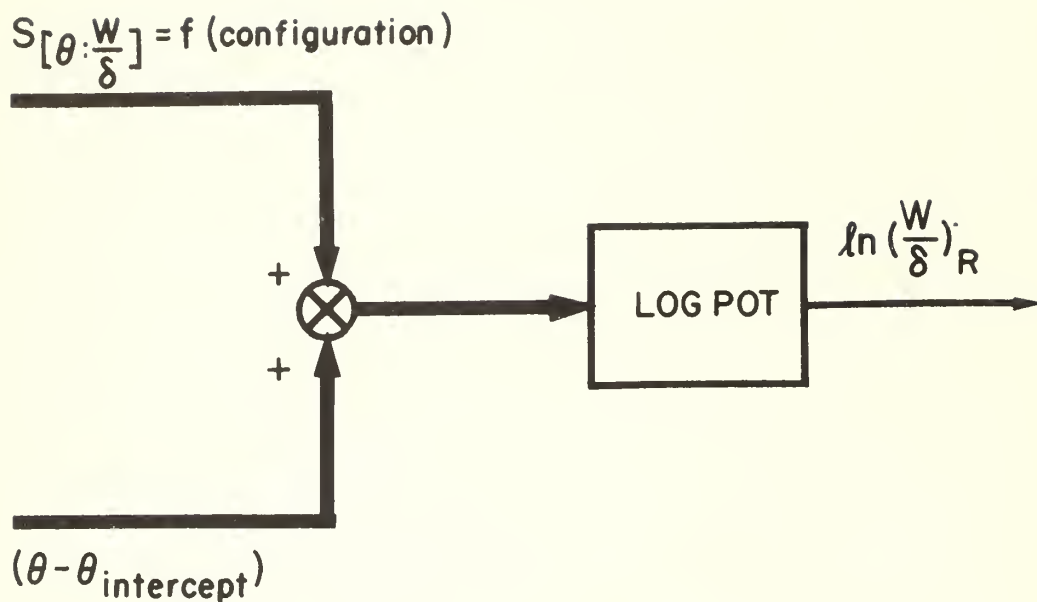


Fig. 5-7 Method of instrumenting  $(\frac{W}{\delta})_R$  curve for various configurations, using approximation of Fig. 5-6.

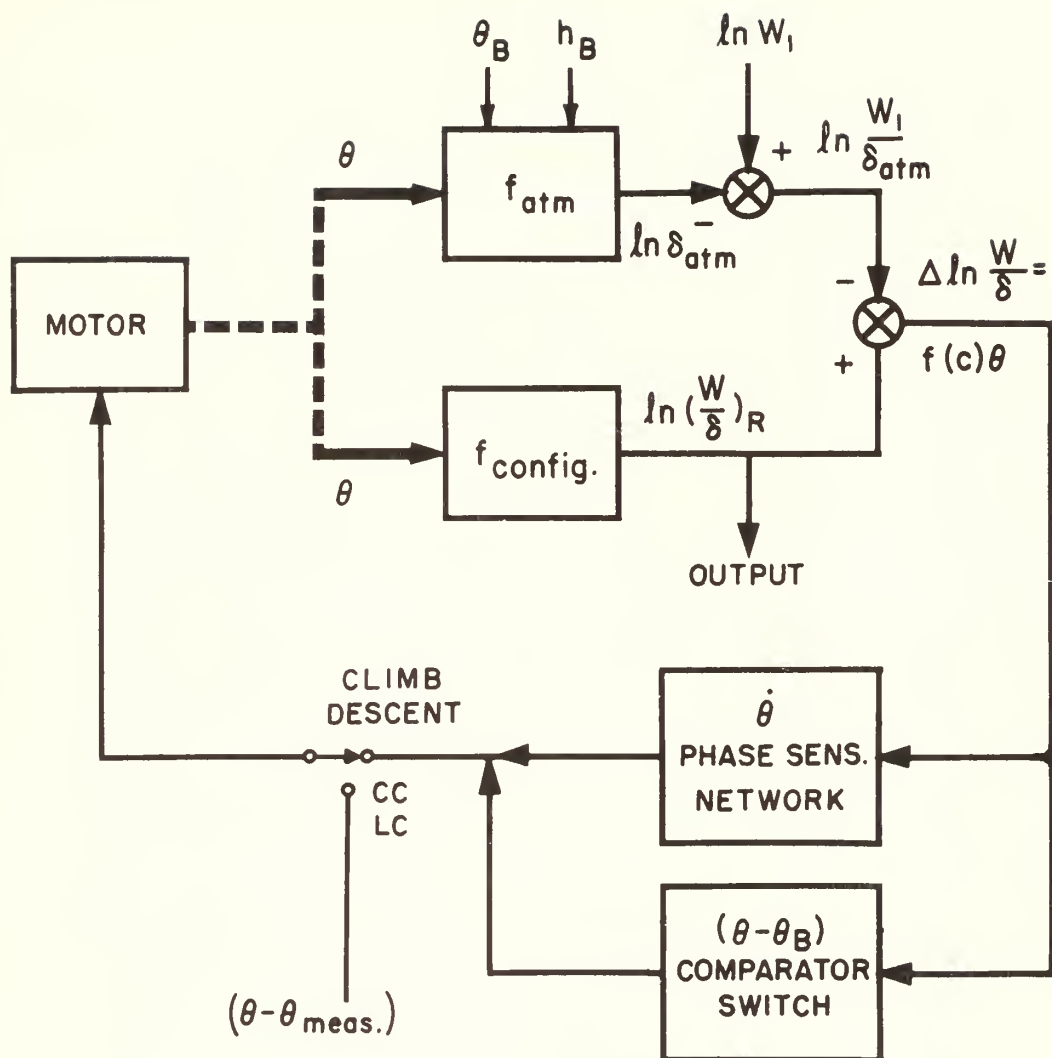


Fig. 5-8 Block diagram of  $(\frac{W}{\delta})_R$  computer, showing use of stored atmospheric data in the climb and descent phases, and measured temperature inputs during cruising flight.

6 to 8 degrees Centigrade. However, it should be pointed out that there is a time lag in using meteorological data. Therefore, it is felt that a direct temperature measurement while at cruising altitude is desirable.

## CHAPTER 6

### INSTRUMENTATION FOR OPTIMUM RANGE MACH NUMBER FOR CRUISING

In this chapter the proposed method of instrumenting the Mach number deviation indicator for cruising flight is indicated.

The method of obtaining the curves of optimum range Mach number vs  $\frac{W}{\delta}$ , Figure (4-2), was outlined in Chapter 4. It will be noted that there is a separate set of curves of  $M_R$  vs  $\frac{W}{\delta}$  with wind as a parameter for every configuration. Using these curves a plot of  $\frac{\partial M_R}{\partial M_W}$  vs  $\frac{W}{\delta}$  can be obtained and is as indicated in Figure 6-1; these curves can be approximated by straight lines and are essentially the same for any configuration. Then for any configuration the optimum range Mach number can be approximated by:

$$\begin{aligned} M_R &= M_{R(\text{no wind})} + \frac{\partial M_R}{\partial M_W} M_W \\ &= M_{R(\text{no wind})} + \left[ (K_{1W} \frac{W}{\delta}) + K_{2W} \right] M_W \quad (6-1) \end{aligned}$$

Where  $K_{1W}$  and  $K_{2W}$  will each have two values, one for a head and one for a tail wind.

A configuration change will change the drag coefficient and thus shift the curve of  $M_R$  vs  $\frac{W}{\delta}$ . This effect of configuration changes is illustrated in Figure 6-2. Examination of Figure 6-2 shows that for normal operating values of  $\frac{W}{\delta}$  the effect of a change in configuration from standard can be accounted for by shifting the  $M_R$  scale left or right. Thus, the effect of configuration change can be accounted for by:



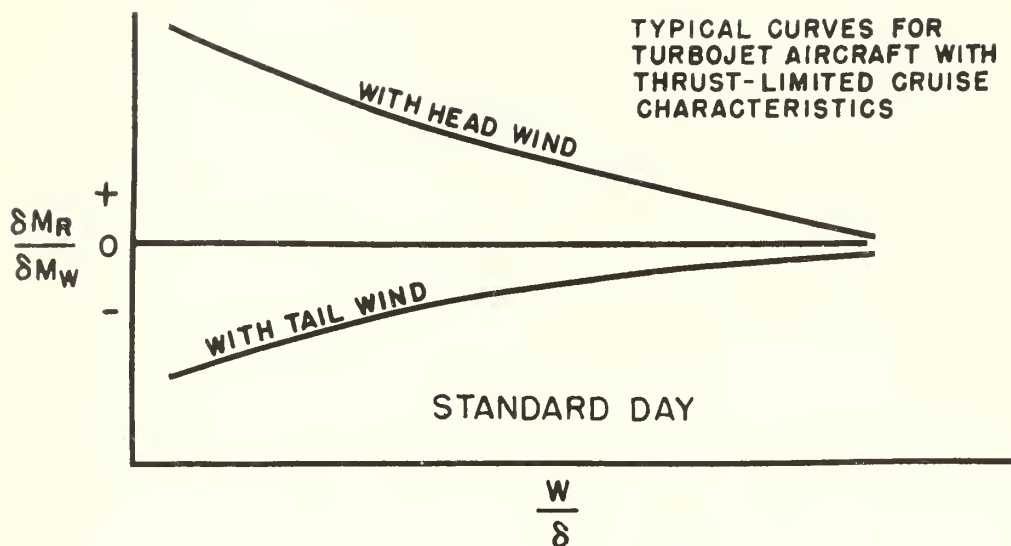


Fig. 6-1 Variation of  $\frac{\partial M_R}{\partial M_W}$  with  $w/\delta$ .

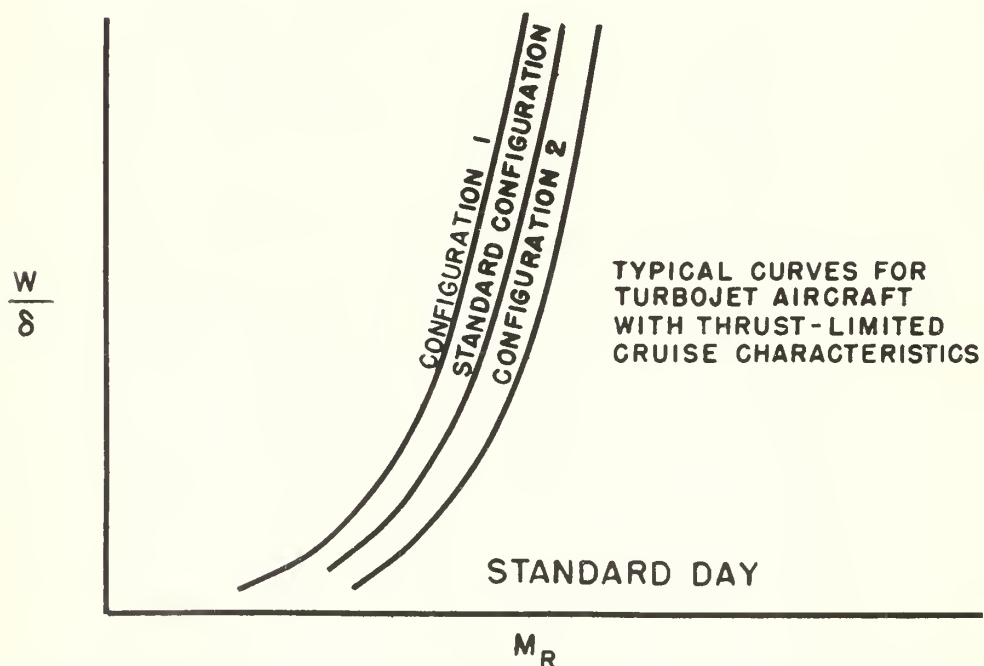


Fig. 6-2 Optimum range Mach number variations with  $w/\delta$  for three configurations.

$$M_R = M_{R(\text{std. config.})} + \Delta M_{R(\text{config.})} \quad (6-2)$$

There will be a different value of  $\Delta M_{R(\text{config.})}$  for each configuration

Plotting the curves of  $M_R$  vs  $\frac{W}{\delta}$  for hot, cold, and standard days indicates that ambient temperature effects are small and can be neglected

Therefore, by storing a single curve of  $M_R$  vs  $\frac{W}{\delta}$  for a standard configuration and no wind condition, the value of  $M_R$  for a given wind and configuration can be derived by:

$$\begin{aligned} M_R &= M_{R(\text{no wind, std. configuration})} \\ &+ \left[ (K_1 W \frac{W}{\delta}) + K_2 W \right] M_W \\ &+ \Delta M_{R(\text{configuration})} \end{aligned} \quad (6-3)$$

The cruising Mach number deviation can be determined by comparing the value of  $M_R$  derived by instrumenting equation 6-3 with the actual instantaneous cruising Mach number of the aircraft. A scheme for instrumenting the airspeed deviation meter during cruising flight is indicated in Figure 6-3.

The same instrumentation for the airspeed deviation indicator, when employing the "RANGE" mode, will be used when cruising at the cruise ceiling (climb cruise path) or cruising at a constant altitude. In the case of the climb cruise path the value of  $M_R$  remains a constant and in the case of the level cruise path  $M_R$  will vary as  $\frac{W}{\delta}$  changes.

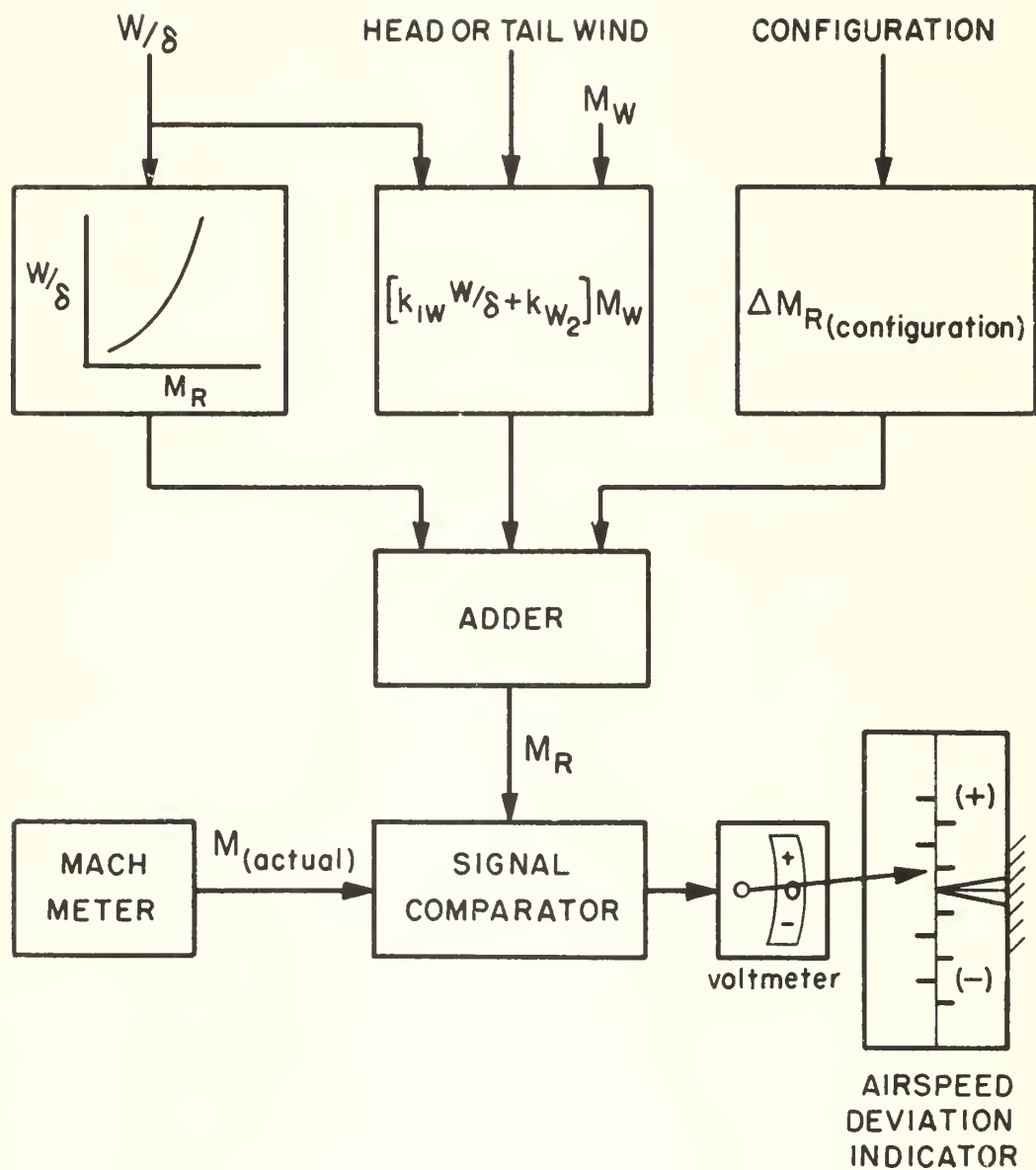


Fig.6-3 Scheme for instrumenting optimum range mach no. deviation for cruising.

## CHAPTER 7

### RANGE IN THE CLIMB CRUISE PHASE

#### I Calculation

The climb cruise phase is the phase in which the airplane is flown according to the conditions for all-out maximum range in cruising flight as outlined in Chapter 3.

Using the same basic curves that were used to plot Figure 3-2, curves of specific range parameter,  $(SR \times \delta)_R$  or  $(\frac{\delta V}{w_f})_R$ , vs  $\frac{W}{\delta}$  can be plotted as shown in Figure 7-1 for each configuration. Ambient temperature effects on these curves are very small, particularly for normal operating values of  $\frac{W}{\delta}$  and therefore will be neglected. (See Appendix B)

It has been shown that flight in the climb cruise phase is at a constant value of  $(\frac{W}{\delta})_R$  for constant  $\theta$ , and that for any given value of  $\frac{W}{\delta}$  there is one cruising Mach number,  $M_R$ , that will give maximum range. Thus, for a given configuration, temperature, and wind the values of  $(\frac{W}{\delta})_R$ ,  $M_R$  and  $(SR \times \delta)_R$ , for the best cruising range are constant if a constant engine efficiency is assumed.

Thus the product:

$$(\frac{W}{\delta})_R \times (SR \times \delta)_R = \frac{VW}{w_f} = SR \times W = \text{Constant} \quad (7-1)$$

Now an expression for range can be written:

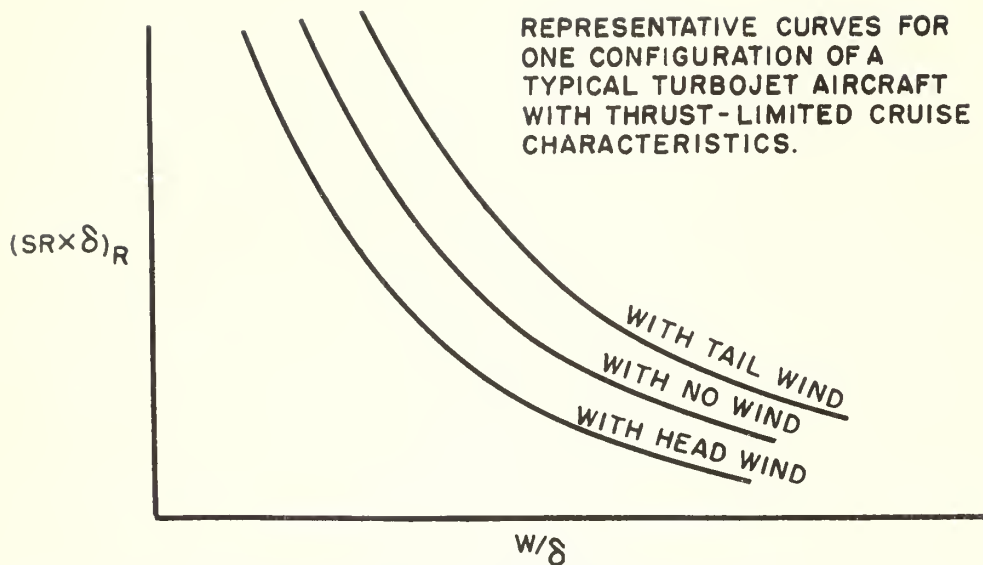


Fig. 7-1 Specific range parameter,  $(SR \times \delta)_R$  variation with  $w/\delta$  for cruising at best range Mach number,  $M_R$ .

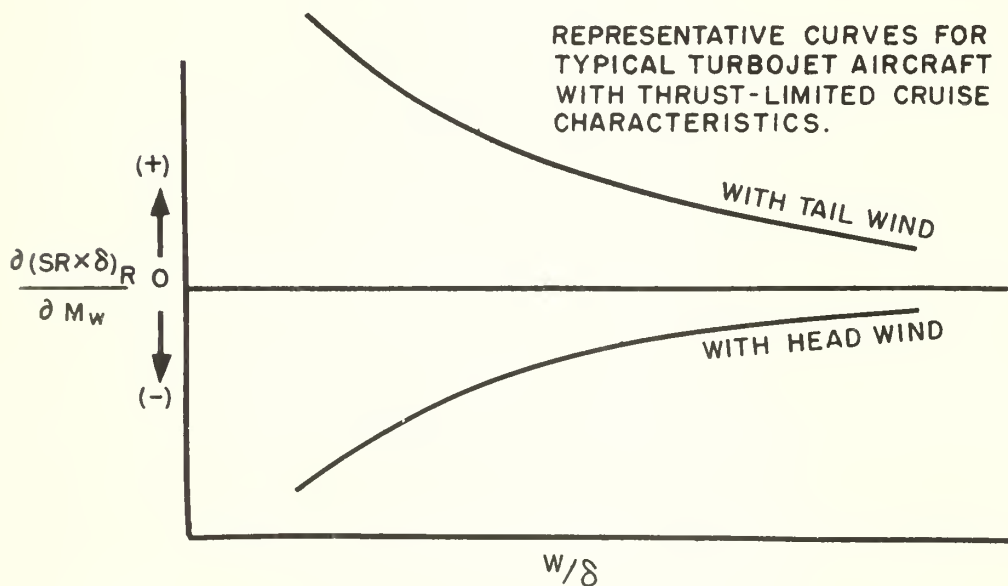


Fig. 7-2 Variation of  $\frac{\partial (SR \times \delta)_R}{\partial M_R}$  with  $w/\delta$ .

$$R_{cc} = - \int_{W_1}^{W_2} (SR) dW \quad (7-2)$$

$$\begin{aligned}
 &= - \int_{W_1}^{W_2} \left( \frac{VW}{w_f} \right)_{R_1} \frac{dW}{W} \\
 &= \left( \frac{VW}{w_f} \right)_{R_1} \ln W \Big|_{W_2}^{W_1} = (SR \times \delta)_R W_1 \ln W \Big|_{W_2}^{W_1} \quad (7-3) \\
 &= \text{Constant} \times \ln W \Big|_{W_2}^{W_1}
 \end{aligned}$$

Where subscript (1) refers to the beginning of the climb cruise phase and subscript (2) to the end of the climb cruise phase.

An alternate form of writing equation (7-3) would be:

$$\begin{aligned}
 R_{cc} &= \left( \frac{\delta V}{w_f} \right)_{R_1} \left( \frac{W}{\delta} \right)_{R_1} \ln \frac{W_1}{W_2} \\
 &= (SR \times \delta)_{R_1} \left( \frac{W}{\delta} \right)_{R_1} \ln \frac{W_1}{W_2} \quad (7-4)
 \end{aligned}$$

$$= (SR)_{R_1} W_1 \ln \frac{W_2 + Q_{cc}}{W_2} \quad (7-5)$$

It should be noted that the above equations hold true only if  $\theta$  remains constant which will not be the case if the starting point of the climb cruise phase is outside the isothermal layer. However, it is not believed that the extra complication in computing  $R_{cc}$ , required to account for variations of  $\theta$  during the climb cruise phase, is justified by the increase in accuracy that could be expected. This is particularly evident when it is

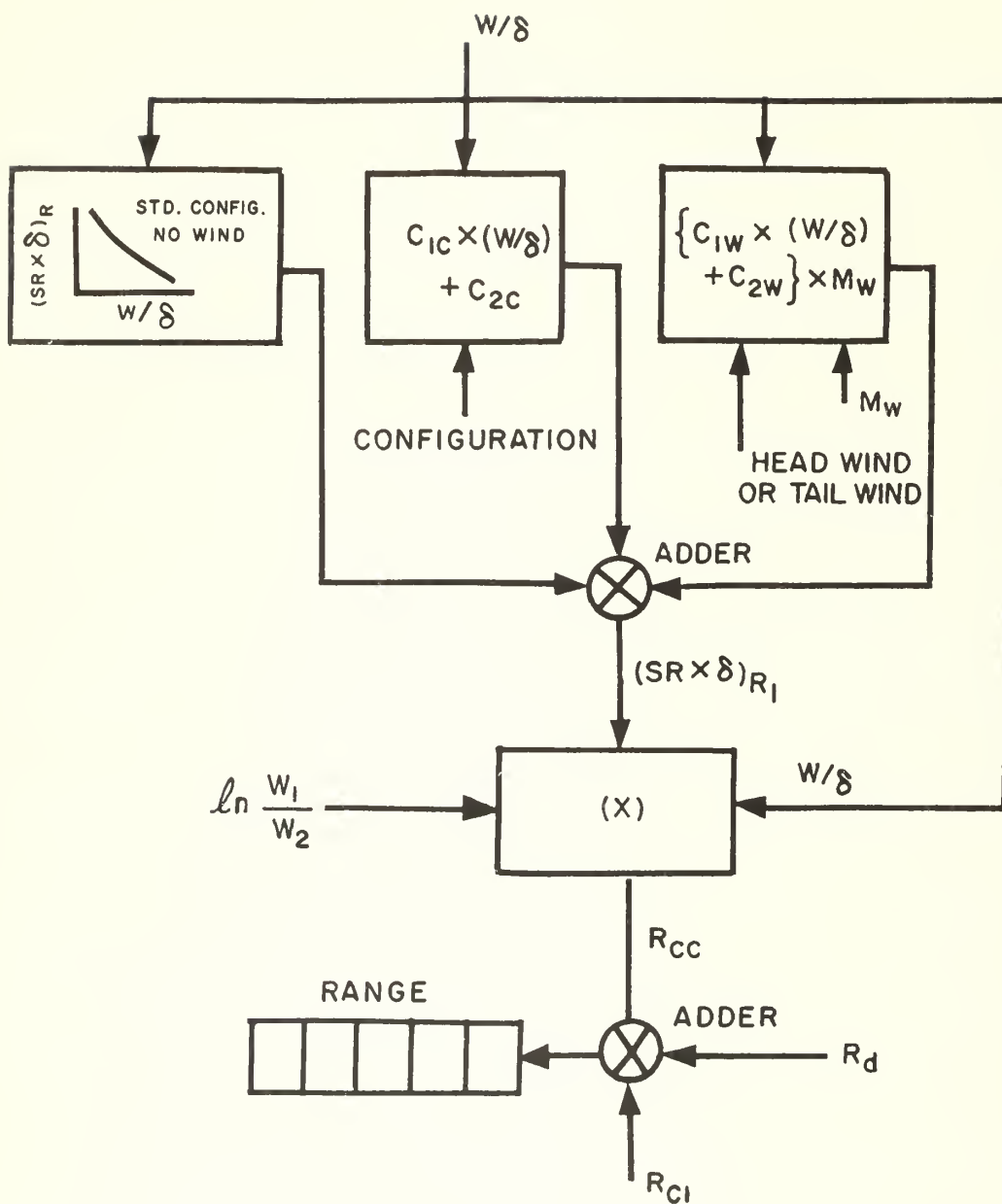


Fig. 7-3 Proposed scheme for predicting range if cruising portion of flight is flown according to optimum range conditions.



observed that the change in  $\theta$  over the climb cruise altitude range would have to be an estimate based on some sort of stored temperature data. Also, the error in predicted range caused by neglecting any change in  $\theta$  during the climb cruise phase would be a conservative error.

## II Instrumentation

From an instrumentation point of view the calculation of range in the climb cruise phase is separated into two divisions: the prediction of the climb cruise range before the climb cruise flight phase is entered, and the prediction of the climb cruise range during flight in this phase.

### A. Prediction Before The Climb Cruise Phase Is Entered

In This phase the predicted value of  $(\frac{W}{\delta})_{R_1}$  is determined as indicated in Chapter 5. With this value of  $(\frac{W}{\delta})_{R_1}$  the value of  $(SR \times \delta)_{R_1}$  can be determined. By instrumenting the curve of  $(SR \times \delta)_R$  vs  $\frac{W}{\delta}$  for the no wind condition and a standard configuration a value of  $(SR \times \delta)_R$  corresponding to  $(\frac{W}{\delta})_{R_1}$  can be derived. Then this value of  $(SR \times \delta)_R$  can be modified for the effects of wind and configuration.

Referring to Figure 7-2 it can be seen that at the higher values of  $\frac{W}{\delta}$  (values at which the climb cruise will normally be flown) the curves of  $\frac{\partial(SR \times \delta)_R}{\partial M_W}$  vs  $\frac{W}{\delta}$  can be approximated by straight lines (one for head wind and one for tail wind). Thus, these curves can be approximated by:

$$\begin{aligned} \frac{\partial(SR \times \delta)_R}{\partial M_W} &= S \left[ \frac{W}{\delta}; \frac{\partial(SR \times \delta)_R}{\partial M_W} \right] \times \frac{W}{\delta} + \text{Constant} \\ &= \text{Constant}_{1W} \times \frac{W}{\delta} + \text{Constant}_{2W} \quad (7-6) \end{aligned}$$

There will be two values for each of the above constants, one for head wind and one for tail wind.

The curve of  $\frac{\Delta (SR \times \delta)_R}{\Delta \text{Configuration}}$  vs  $\frac{W}{\delta}$  is similar to the curve of  $\frac{\partial (SR \times \delta)_R}{\partial M_W}$  vs  $\frac{W}{\delta}$ . Thus, making the same straight line approximation, the correction for configuration change can be written:

$$\Delta (SR \times \delta)_R = (\text{Constant}_{1C} \times \frac{W}{\delta}) + \text{Constant}_{2C} \quad (7-7)$$

There will then be different values for the above constants for each different configuration.

Now the predicted value of  $(SR \times \delta)_{R_1}$  in this phase can be approximated by instrumenting the following formula:

$$\begin{aligned} (SR \times \delta)_{R_1} &= (SR \times \delta)_{R_1} \text{ (no wind-std. config.)} \\ &+ \left\{ \left[ \text{Constant}_{1W} \times \left( \frac{W}{\delta} \right)_{R_1} \right] + \text{Constant}_{2W} \right\} M_W \\ &+ \left\{ \left[ \text{Constant}_{1C} \times \left( \frac{W}{\delta} \right)_R \right] + \text{Constant}_{2C} \right\} \quad (7-8) \end{aligned}$$

Then using the value of  $(SR \times \delta)_{R_1}$  derived from equation (7-8), the predicted climb cruise range can be derived by instrumenting:

$$R_{cc} = (SR \times \delta)_{R_1} \left( \frac{W}{\delta} \right)_{R_1} \ln \frac{W_1}{W_2} \quad (7-4)$$

A scheme for instrumenting the predicted range before the climb cruise phase is entered is shown in Figure 7-3. This system will be in operation when the "RANGE" mode has been selected and the "CLIMB" flight path selector switch depressed.

The preceeding proposed instrumentation to predict range would be employed when the quantity of fuel for cruising is sufficiently large that the maximum range is achieved by climbing

to and cruising at the optimum value of  $(\frac{W}{\delta})_R$ . The minimum fuel problem where the maximum range is achieved by climbing to and cruising at some  $\frac{W}{\delta}$  below the cruise ceiling will be discussed in Chapter 13.

#### B. Prediction Of The Climb Cruise Range During Flight In This Phase

The predicted climb cruise range while flying at the  $(\frac{W}{\delta})_R$  and  $M_R$  indicated for the climb cruise phase will be based on the actual value of  $\frac{W}{\delta}$ . The same instrumentation indicated in Figure 7-3 can be used. The value of  $Q_{cl}$  and  $R_{cl}$  will be zero and  $\frac{W}{\delta}$  will be the actual value based on actual  $\delta$  and weight.

### III Methods Of Using Predicted Climb Cruise Ranges

The proposed method of instrumenting the predicted climb cruise range permits considerable flexibility in its employment by the pilot. Following are some examples of ways in which this predicted cruising range can be employed.

On any type of flight the pilot is able to set in the reserve fuel quantity desired at the completion of cruising and then have an indication of what his maximum range will be. Or, the pilot may, by manipulating the reserve fuel input, select a given cruising range and then read the value of the reserve fuel quantity he can expect to have for such a cruising range.

On a tactical mission over a target at a known distance from the final landing location the pilot could select the desired reserve fuel quantity, depress the "CLIMB" button on the display control panel and then remain in the target area until the indicated range remaining equaled the distance to home base. This feature in the case of close support or target of opportunity type missions would be particularly desirable; for, it would enable the pilot to make a more intelligent determination of the number of runs to make in expending his ordnance. In the case of aerial combat it would indicate at what point he must break off contact in

order to have sufficient fuel to return to home base.

On missions where the aircraft is under positive control of a ground controller the range indicator would enable the pilot to give accurate information to the controller on his range remaining.

A possible means of increasing the usefulness of the instrument would be to incorporate a "break-away range" setting which would be set in by the pilot. This "break-away range" could be continuously compared electrically to the predicted range; and when the two become equal, a "break-away" indicator light could be actuated. In effect, this warns the pilot when to head back to base to arrive there with the selected amount of reserve fuel.

## CHAPTER 8

### RANGE FOR LEVEL CRUISE

In this chapter the problem of maximum range for a constant altitude cruise path will be discussed. When the pilot selects the "RANGE" mode and depresses the "LEVEL CRUISE" flight path selector switch, the Mach number deviation indicator will indicate the difference between the present Mach number and the optimum range Mach number,  $M_R$ , for cruising at the present altitude. Also, the indicated range and endurance remaining will be based on cruising at the present altitude and indicated  $M_R$ , and on a minimum time descent. The altitude deviation indicator could be blanked off or could indicate the deviation from the cruise ceiling; for, the pilot would refer to the aircraft's altimeter in maintaining altitude.

The cruising range can be determined by:

$$R_{lc} = (SR \times \delta)_{R_{average}} \times \frac{Q_{lc}}{\delta} \quad (8-1)$$

To use this formula the average value of  $(SR \times \delta)_R$  must be determined.

A proposed method of determining  $(SR \times \delta)_{R_{average}}$  is:

$$\begin{aligned} (SR \times \delta)_{R_{average}} &= (SR \times \delta)_{R_{instantaneous}} \\ &\times \left[ 1 + k_{lc} Q_{lc} \right] \end{aligned} \quad (8-2)$$

To use this formula the value of  $k_{lc}$  must be determined. Two possible methods for calculating  $k_{lc}$  are:

1. Calculate a single value of  $k_{lc}$  valid over the normal range of  $\frac{W}{\delta}$  that is expected to be covered while cruising. This can be done by examining the  $(SR \times \delta)_R$  vs  $\frac{W}{\delta}$  curve and calculating  $k_{lc}$  as indicated in Figure 8-1.
2. Calculate a value of  $k_{lc}$  to be used in equation (8-1) that is a function of  $(\frac{W}{\delta})_{inst}$  and weight of fuel remaining.

The second method of calculating  $k_{lc}$  should give more accurate results when calculating  $(SR \times \delta)_R$  average. However, the first method should give sufficiently accurate results, particularly if use of the constant altitude cruise is restricted to reasonably high altitudes. This restriction is not unreasonable; for, any normal cruising altitude for a turbojet aircraft is reasonably high. A suggested minimum altitude would be in the vicinity of 25,000 feet. The instrument could continue to function below the specified minimum altitude, but the error in the indicated range and endurance would increase with decreasing altitudes.

The instantaneous value of  $(SR \times \delta)_{R_1}$  will be derived by entering the stored curve of  $(SR \times \delta)_R$  vs.  $\frac{W}{\delta}$  with the actual instantaneous value of  $\frac{W}{\delta}$ . Then the value of  $(SR \times \delta)_R$  will be corrected for wind and configuration as indicated in Chapter 7 by using the formula:

$$\begin{aligned}
 (SR \times \delta)_{R_1} = & (SR \times \delta)_{R_1 \text{ (no wind - std. config.)}} \\
 & + \left\{ \left[ \text{Constant}_{1W} \times \left( \frac{W}{\delta} \right)_{inst} \right] + \text{Constant}_{2W} \right\} M_W \\
 & + \left\{ \left[ \text{Constant}_{1C} \times \left( \frac{W}{\delta} \right)_{inst} \right] + \text{Constant}_{2C} \right\} \quad (8-3)
 \end{aligned}$$

The optimum cruising speed for maximum range in level cruise would be instrumented the same as for the maximum range climb cruise as indicated in Chapter 6.



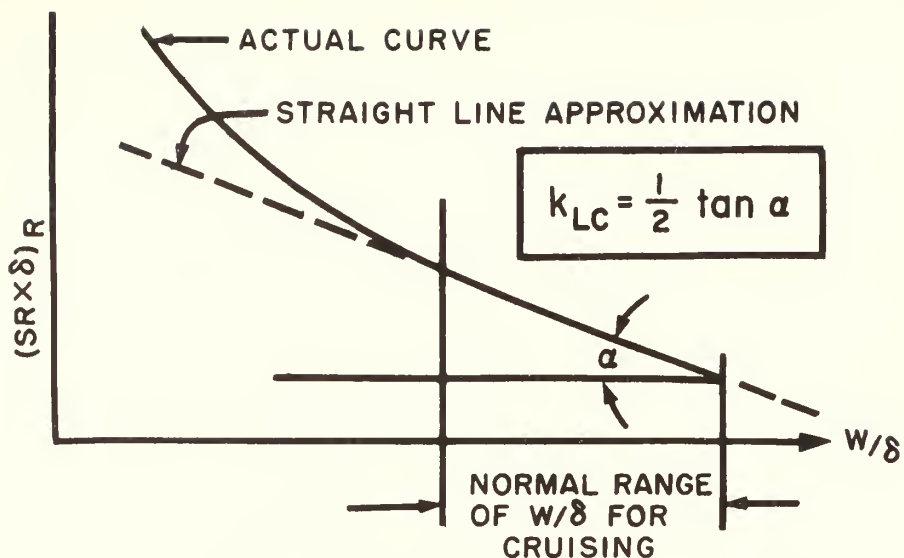


Fig. 8-1 Method for determining value of  $k_{LC}$ .

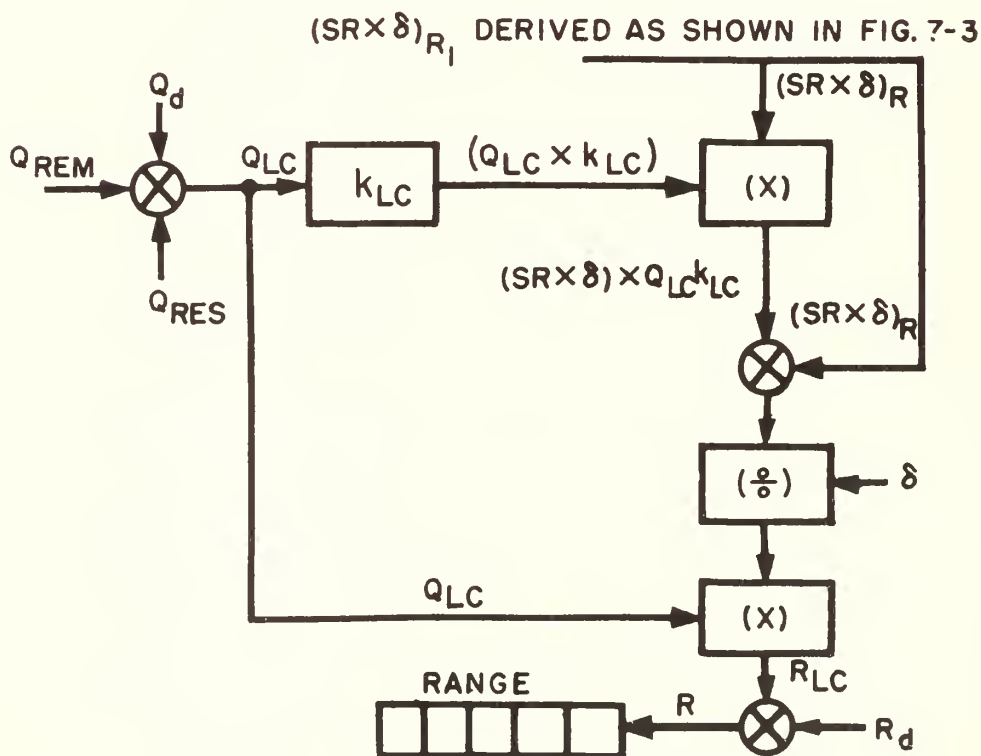


Fig. 8-2 Proposed scheme for obtaining level cruise range.



The method of instrumenting the range indicator for level cruise by employing formula (8-3), formula (8-2) and:

$$R_{lc} = (SR \times \delta) R_{average} \times \frac{Q_{lc}}{\delta} \quad (8-1)$$

is indicated in Figure 8-2.

Incorporation of this "level cruise" feature in the instrument will permit the pilot to know his status as far as range and endurance remaining is concerned if he must maintain a given altitude. It will also give a basis for making or requesting in-flight flight plan changes. However, this feature can not be used for preflight planning for the indicated range, and endurance is always based on a constant altitude flight at present altitude only.

## CHAPTER 9

### RANGE IN THE CLIMB PHASE

Analysis of climb patterns indicates that for a typical turbo-jet aircraft with thrust-limited cruise characteristics a military power climb to cruising altitude gives the best maximum overall range, or maximum overall endurance, depending on the mode selected.

The range covered in the climb phase, during a military power climb at the most efficient airspeed for maximum range or endurance, may be plotted versus the pressure ratio (with weight as a parameter) as shown in Figure 9-1. It is evident that a good approximation to the range may be easily obtained by correcting the curve for an average weight, by means of a multiplicative factor to account for the weight perturbation. Other perturbations may be treated similarly. Thus, beginning with the assumptions:

$$R_{cl} = f(W, \theta_{cr}, M_W, \text{config}, + \dots) \quad (9-1)$$

and,

$$\Delta R_{cl} = R_{cl\_actual} - R_{cl\_ref} \quad (9-2)$$

$$\begin{aligned} \Delta R_d = & \frac{\partial R_{cl}}{\partial W} \Delta W + \frac{\partial R_{cl}}{\partial \theta_{cr}} \Delta \theta_{cr} + \frac{\partial R_{cl}}{\partial M_W} \Delta M_W \\ & + \frac{\partial R_{cl}}{\partial \text{config}} \Delta \text{config} + \dots \quad (9-3) \end{aligned}$$

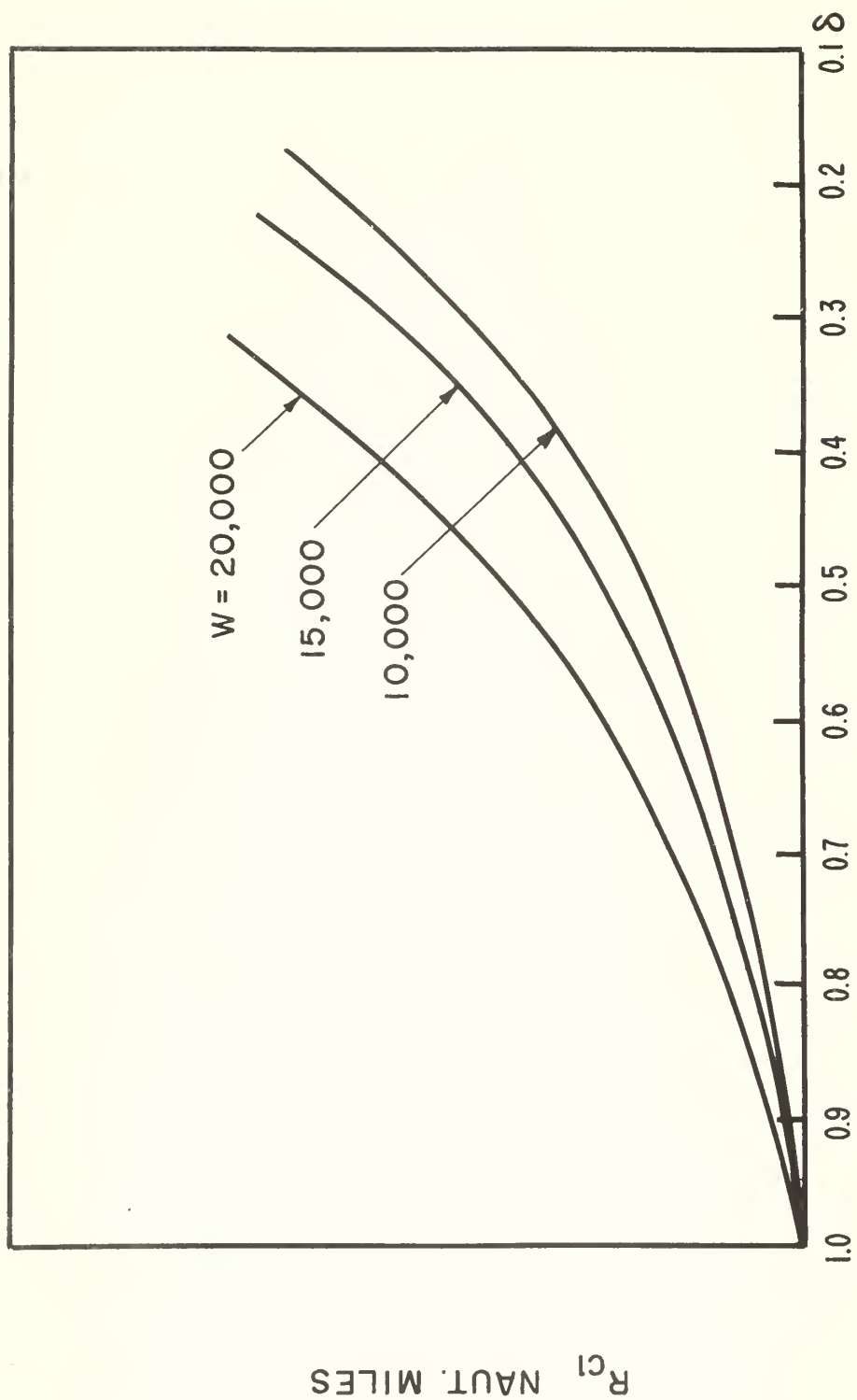


Fig. 9-1 Typical curve of range covered in climb vs. pressure ratio,  $\delta$ .

and rearranging equations (9-2) and (9-3), there results:

$$R_{cl} = R_{cl_{ref}} + \frac{\partial R_{cl}}{\partial W} \Delta W + \frac{\partial R_{cl}}{\partial \theta_{cr}} \Delta \theta_{cr} + \frac{\partial R_{cl}}{\partial M_W} \Delta M_W + \frac{\partial R_{cl}}{\partial \text{config}} \Delta \text{config} + \dots \quad (9-4)$$

In the foregoing development  $R_{cl}$  is defined as the total range covered from the beginning of a sea-level military power climb to the cruising altitude which corresponds to the pressure ratio  $\delta_1$ . In equations (9-2) and (9-4)  $R_{cl_{ref}}$  represents the value of  $R_{cl}$  of the basic stored curve, evaluated at the particular value  $\delta_1$ . The first three partial derivative terms are multiplied by the appropriate increment. The entire last term involving configuration would most conveniently be applied as one of a finite number of multiplicative factors, each representing the correction factor necessary to obtain the range value for the configuration in question from the average reference range figure. Fuel consumed in climb and endurance in climb may be expressed in a similar manner.

It is necessary to account for cases in which the climb actually starts from some  $\delta = \delta_i$  which is an initial intermediate value of  $\delta$  other than the sea level value and ends at  $\delta_1$  which occurs at the beginning of the cruise phase. The actual range covered from the initial altitude to the cruise altitude may be found in the following manner: Subtract the range covered in a climb from sea level to the intermediate altitude, from the range covered in a climb from sea level to the cruising altitude, i. e.:

$$R_{cl_{i-1}} = R_{cl_1} - R_{cl_i} \quad (9-5)$$

In order to compute the weight  $W_1$  at the end of climb ( $Q_{cl} - Q_i$ ), the weight of fuel used in climbing, must be subtracted from  $W_i$ . (This value of weight will be used in the prediction

phase, in determining the cruising altitude, range and endurance).

The optimum airspeed in the climb phase is predominantly a function of pressure altitude. For some tactical jet aircraft, there is negligible dependence on aircraft weight, wind, configuration and atmospheric temperature. For others there will be some dependence on these parameters. Therefore, a basic climb airspeed curve may be stored in the computer, in the form of Mach number versus pressure ratio. If there be sufficient dependence on other parameters such as weight, wind, and configuration, the basic curve may be modified to account for these parameters. Furthermore, the airspeed schedule for maximum endurance is almost the same as for maximum range, so that the shift from maximum endurance airspeed schedule might also be treated as a small perturbation of the basic curve. In some cases the difference is negligible and the two airspeed schedules may be stored as a single curve and no corrections applied

## CHAPTER 10

### RANGE IN DESCENT PHASE

Three descent patterns, in order of effectiveness from a range standpoint, are:

- a. Optimum glide path (engine off) - (low altitude relight).
- b. Optimum idle rpm descent
- c. Minimum time descent at idle rpm

For computer simplification, only one of the above descents is felt to be necessary as a basis on which to compute range and endurance values. The first technique, in (a) above, while yielding the maximum range for a given amount of fuel, is in the category of an emergency technique and would not normally be used operationally. The second pattern, in (b) above, is desirable in some cases where maximum range without shutting down the engine and relighting must be obtained. The most common descent technique from an operational standpoint is that given in (c) above. Therefore, it is this descent that has been chosen as a basis for computer instrumentation. The minimum time descent airspeed schedule will be stored and available to the pilot when he depresses the "IDLE DESCENT" button.

In the computer display, as described in Chapter 3, an indication of the time to commence an optimum range idle rpm descent to arrive at sea level with the preset fuel reserve has been proposed. This feature alone would be relatively simple to instrument and might be accomplished by means of a fuel

quantity comparator, completing the indicator light circuit when  $Q_{rem} \leq Q_d + Q_{res}$ . An extension of this feature might be to store, in addition, the airspeed schedule for such an idle rpm descent.

The range covered in a minimum time descent is principally a function of the initial pressure ratio at which the descent is commenced. Figure 10-1 shows a typical curve of  $R_d$  versus  $\delta$ , with configuration as a parameter. Wind also affects range in the descent, although to a lesser degree. Wind effect might be considered in the instrumentation by adding a correction, which would be a function of wind velocity and time in descent.

There appears to be almost negligible dependence of range on other parameters such as weight, configuration, etc.

Considering that only a small portion of the flight is devoted to the minimum time descent the storing of a single range in descent versus initial pressure ratio curve is proposed. This curve may be corrected for wind effect. Thus, an approximation to the range in descent may be written:

$$R_{d_{\delta_2}} = R_{d_{\delta_{2_{ref}}}} + k E_d V_W \quad (10-1)$$

in which changes in configuration, weight, etc., are neglected.

The endurance in descent,  $E_d$ , and weight of fuel consumed in descent,  $Q_d$ , may be treated in a similar manner. In fact, to use Equation (10-1) it is necessary to know  $E_d$ .

In order to use the stored curves mentioned above to predict the range and endurance in descent it is necessary to know the value of  $\delta_2$ , the pressure ratio at end of cruise phase. In the case of the level cruise path this is a simple matter because  $\delta_2 = \delta_1$ . In the case of the climb cruise path  $(\frac{W}{\delta})_R = \text{constant} = \frac{W_2}{\delta_2}$ . Employing this fact  $\delta_2$  may be computed by:



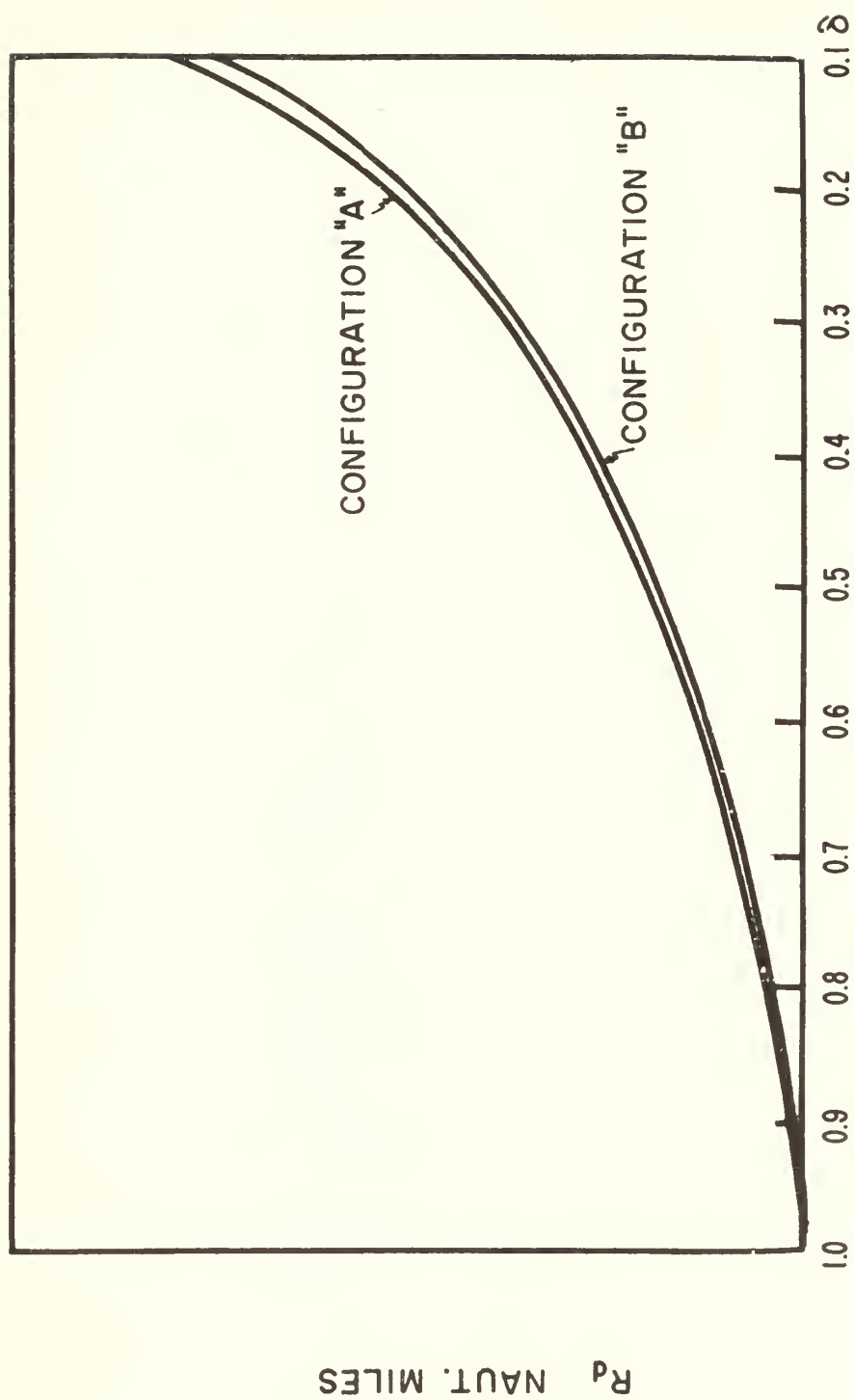


Fig. 10-1 Range covered in idle rpm minimum time descent as a function of initial pressure ratio,  $\delta_2$ , (typical case).

$$\delta_2 = \frac{W_2}{\left(\frac{W}{\delta}\right)_R} = \frac{W_1 - Q_{\text{rem}} + Q_d + Q_{\text{res}}}{\left(\frac{W}{\delta}\right)_R} \quad (10-2)$$

From equation (10-2) it is seen that the value of  $Q_d$  must be determined in order to calculate  $\delta_2$ . But,  $\delta_2$  must be known if  $Q_d$  is to be determined from the curve of  $Q_d$  vs  $\delta_2$ . Therefore, an approximation method must be used to determine  $\delta_2$ . Following are three possible methods for getting an approximate value of  $\delta_2$ :

1.  $Q_d$  is small compared to  $W_1$  thus an initial value of  $\delta_2$  may be determined by:

$$\delta_2 = \frac{W_1 - Q_{\text{rem}} + Q_{\text{res}}}{\left(\frac{W}{\delta}\right)_R} \quad (10-3)$$

Then with the value of  $\delta_2$  derived from equation (10-3) a value of  $Q_d$  may be obtained from the curve of  $Q_d$  vs  $\delta_2$ . Finally, using this value of  $Q_d$  equation (10-2) can be employed to determine a second and more accurate value of  $\delta_2$ .

2. Noting that  $Q_d$  is small, equation (10-2) could be employed to compute  $\delta_2$  by replacing  $Q_d$  by a constant. This method is probably the simplest and most desirable to instrument, and should be sufficiently accurate.
3. Again noting that  $Q_d$  is small, equation (10-3) could be used alone to determine  $\delta_2$ . This method is almost the equivalent of 2. in simplicity but not quite as accurate, since it neglects  $Q_d$  entirely.

## CHAPTER 11

### TOTAL RANGE IN RANGE MODE

The total range prediction may be divided into three phases:

a. Pre-Cruise Phase (Before Reaching Cruise Altitude)

The component phases of range which must be added to give total range are range in climb, range in cruise, and range in descent. These are discussed at length in Chapters 7, 8, 9, and 10. A simple adder circuit may be used to give the total range figure, as shown in Figure 11-1.

b. Cruise Phase At Cruise Altitude

The total range when flying at the cruise altitude is merely the sum of the cruise range and descent range. In this case the same adder can be employed since  $R_{cl}$  is zero when the "CLIMB CRUISE" or "LEVEL CRUISE" flight selector switch is depressed.

c. Descent Phase

The possibility of a descent occurring in mid-flight, followed by subsequent climb, cruise, and final descent phase must not be overlooked. For this reason, the range prediction while in the descent phase should be based upon actual available fuel,  $Q_{rem} - Q_{res}$ , and not upon the stored descent curves alone.

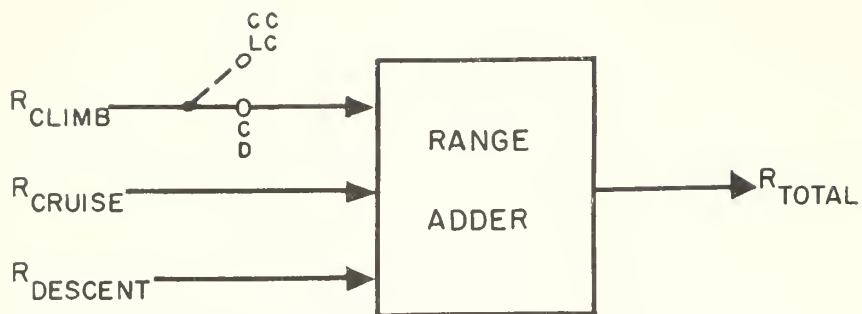


Fig. 11-1 Total range adder

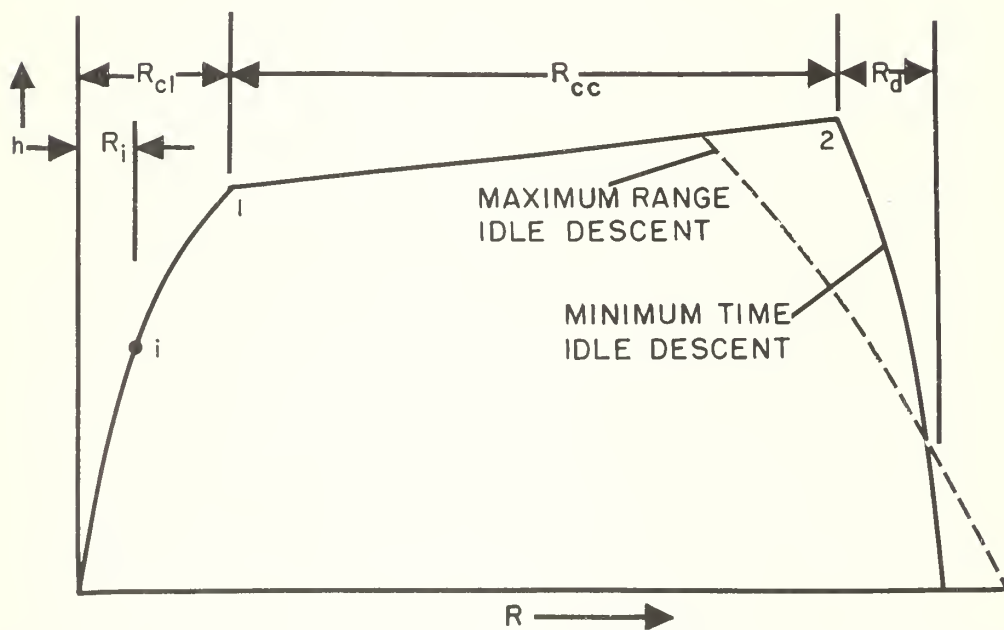


Fig. 11-2 Typical flight profile, constant  $\left(\frac{W}{S}\right)_R$  constant  $M_R$  cruise, showing range covered in various phases

If the descent curves alone were employed, and a mid-flight descent were initiated, the range and endurance predictions would be extremely pessimistic; for, they would only apply to the small remainder of the descent phase and would not account for the additional capability of another complete flight profile as shown in Figure 11-2. Therefore, pushing the "IDLE DESCENT" selector should actuate the same range and endurance prediction portions of the computer as though the "CLIMB" selector were pushed in. The only difference would be that the minimum time idle rpm descent airspeed schedule would be presented on the airspeed deviation meter.



## CHAPTER 12

### ENDURANCE

In this chapter the instrumentation of the endurance features of the proposed instrument will be discussed. A complete study of how the endurance features should be handled will not be attempted.

The endurance problems can be divided into two main subdivisions:

#### 1. The Problems Encountered When The "RANGE" Mode Has Been Selected

In the "RANGE" mode the endurance indicator must be instrumented to give the endurance when any of the flight path selector switches has been selected.

- a. When the "LEVEL CRUISE" switch is depressed the endurance could be derived from:

$$E = E_{cl} + E_{cruise} + E_d \quad (12-1)$$

In this equation  $E_{cl}$  and  $E_d$  will be determined from stored curves and  $E_{cruise}$  for level cruising can be determined by:

$$E_{lc} = \frac{R_{lc}}{(M_{R_{(average)}} \pm M_W) \times \text{Constant} \times \sqrt{\theta}} \quad (12-2)$$

In equation (12-2)  $R_{lc}$  will be determined as indicated in Chapter 8,  $\theta$  will be from measured data (or stored data if no



means can be provided for accurately measuring temperature), and  $M_{R(\text{average})}$  can be approximated by averaging the instantaneous value of  $M_R$  with the value of  $M_{R_2}$  derived by entering the  $M_R$  vs  $\frac{W}{\delta}$  curve with the value of  $(\frac{W}{\delta})_2$ .

- b. When the "CLIMB CRUISE" switch is depressed the endurance can be computed by using equation (12-1). In this case  $E_{\text{cruise}}$  will be  $E_{cc}$  which can be approximated by equation (12-2). In employing equation (12-2)  $M_{R(\text{average})}$  will simply be the cruising  $M_R$  which is a constant. The value of  $\theta$  can be taken as the instantaneous value. It is recognized that using the instantaneous value of  $\theta$  in formula (12-2) will introduce an error; however, the maximum error that could conceivably be introduced by using this value of  $\theta$  was found to be on the order of 6 or 7 percent for a typical turbojet aircraft with thrust-limited cruise characteristics. This error would be that encountered at the beginning of the climb cruise phase for a maximum-range-minimum-reserve fuel flight where the entire climb cruise phase was accomplished at altitudes below the isothermal temperature level. Thus the error that would be encountered under normal circumstances in which a portion of the flight is made in the isothermal layer would be considerably smaller. Also, the percent error would decrease as the flight progressed. A method that could be used to reduce this error would be to deliberately introduce a percentage error in the opposite sense to that caused by using the instantaneous value of  $\theta$  in equation (12-2). This could be accomplished by simply multiplying

$E_{cc}$  by a factor which would reduce the maximum possible error but would also establish a minimum possible error.

- c. When the "CLIMB" switch is depressed the endurance would be derived by the same method as indicated for "CLIMB CRUISE" above except the value of  $\theta$  to be used in equation (12-2) would be the estimated value of  $\theta$  derived as indicated in Chapter 5.
- d. When the "DESCENT" switch is depressed the endurance will read the same as if the "CLIMB" switch had been depressed.

## 2. The Problems Encountered When In The "ENDURANCE" Mode Has Been Selected

In the "ENDURANCE" mode the flight parameters to be employed are those associated with a minimum value of fuel flow. Fuel flow performance curves indicate that fuel flow decreases with velocity so that the stall speed would be the speed for minimum fuel flow. The stall speed is approximately the speed for the maximum lift-to-drag ratio; thus, this criterion could be used for maximum endurance with possibly a factor added to the speed to increase flying stability.

A curve of specific endurance parameters versus  $(\frac{W}{\delta})_E$  may be obtained, analogous to specific range parameter versus  $(\frac{W}{\delta})_R$  curve and the "ENDURANCE" mode instrumented in a manner analogous to that employed for the "RANGE" mode. In some respects the maximum endurance problem is simpler than the maximum range problem. For example, corrections for wind need not be applied, as maximum endurance flight is independent of wind.



## CHAPTER 13

### THE MINIMUM FUEL PROBLEM IN THE RANGE MODE

The minimum fuel problem in the range mode arises when the total fuel available for the flight ( $Q_{\text{rem}} - Q_{\text{res}}$ ) is insufficient to justify climbing to  $(\frac{W}{\delta})_R$ . Instead, the maximum possible range will be obtained by climbing to, and cruising at, some smaller value of  $\frac{W}{\delta}$ , which will be termed  $(\frac{W}{\delta})_{R_{\text{mf}}}$ . From studies of typical aircraft, the fuel remaining must be rather small, (less than about 20% of the total fuel capacity).

In order to account for fuel consumed in climbing to the initial altitude, a value of  $Q_1$  must be added to the available fuel to form a basis for determining the value of  $(\frac{W}{\delta})_{R_{\text{mf}}}$ . A typical plot of  $(\frac{W}{\delta})_{R_{\text{mf}}}$  vs  $(Q_{\text{rem}} - Q_{\text{res}} + Q_1)$  is shown in Figure 13-1.

A possible scheme of handling the minimum fuel problem would be to use a fuel quantity comparator to compare  $(Q_{\text{rem}} - Q_{\text{res}} + Q_1)_{\text{actual}}$  with  $(Q_{\text{rem}} - Q_{\text{res}} + Q_1)_{\text{min for } (\frac{W}{\delta})_R}$ . If the fuel quantity is very low and the minimum fuel problem exists, then a warning light of some type (not shown in Figure 3-1) might be used to inform the pilot of this condition when the "CLIMB" switch is depressed. A recommended display would consist of having the words "MINIMUM FUEL" appear in lighted letters on the altitude deviation indicator. (When this warning light comes on, the pilot may be able to eliminate the minimum fuel situation by decreasing the fuel reserve setting, thus increasing the value of  $(Q_{\text{rem}} - Q_{\text{res}} + Q_1)$ , if he so desires). However, assuming the fuel reserve setting remains undisturbed,

and the "CLIMB" flight path selector switch is depressed, the altitude deviation meter (with the words "MINIMUM FUEL" lighted) would show the deviation of  $(\frac{W}{\delta})$  from a reference value of  $(\frac{W}{\delta})_{R_{mf}}$  taken from the stored curve of Figure 13-1 and the indicated predicted Range would be:  $R_{cl} = R_{cl_i} + R_{lc} + R_d$ . Each of these range values can be determined as indicated in previous chapters.

The pilot then follows the optimum climb schedule until the altitude deviation indicator reads zero. Then the pilot will depress the "LEVEL-CRUISE" flight path selector switch and follow the normal level-cruise technique; that is, the pilot maintains altitude by means of the altimeter and he maintains the optimum airspeed for maximum range at the present altitude by using the airspeed deviation meter. The range computation is now resolved into a level-cruise phase and a descent phase, and computed in the same manner as described in Chapter 8.

Since the entire flight, and hence the cruise phase itself, will be of short duration, it is proposed that the cruise phase be flown at a constant altitude instead of at constant  $(\frac{W}{\delta})_{R_{mf}}$ . A negligible adverse effect on range would result, due to the short time involved, and instrumentation is simplified considerably. The advantage in this choice is that the "LEVEL-CRUISE" flight path selector switch and all the associated instrumentation may be employed. The only change that may be necessary is an adjustment in  $k_{lc}$  of equation (8-2) to account for the lower cruising altitudes.

In order to fly the constant  $(\frac{W}{\delta})_{R_{mf}}$ , constant  $M_{R_{mf}}$ , type of cruise some additional instrumentation would be required since the  $(\frac{W}{\delta})_R$  section of the computer would have to be modified to hold  $(\frac{W}{\delta})_{R_{mf}}$  as a reference value. This additional instrumentation is not considered justified.

A minimum fuel problem also exists for the endurance mode and would have to be instrumented in a similar manner.

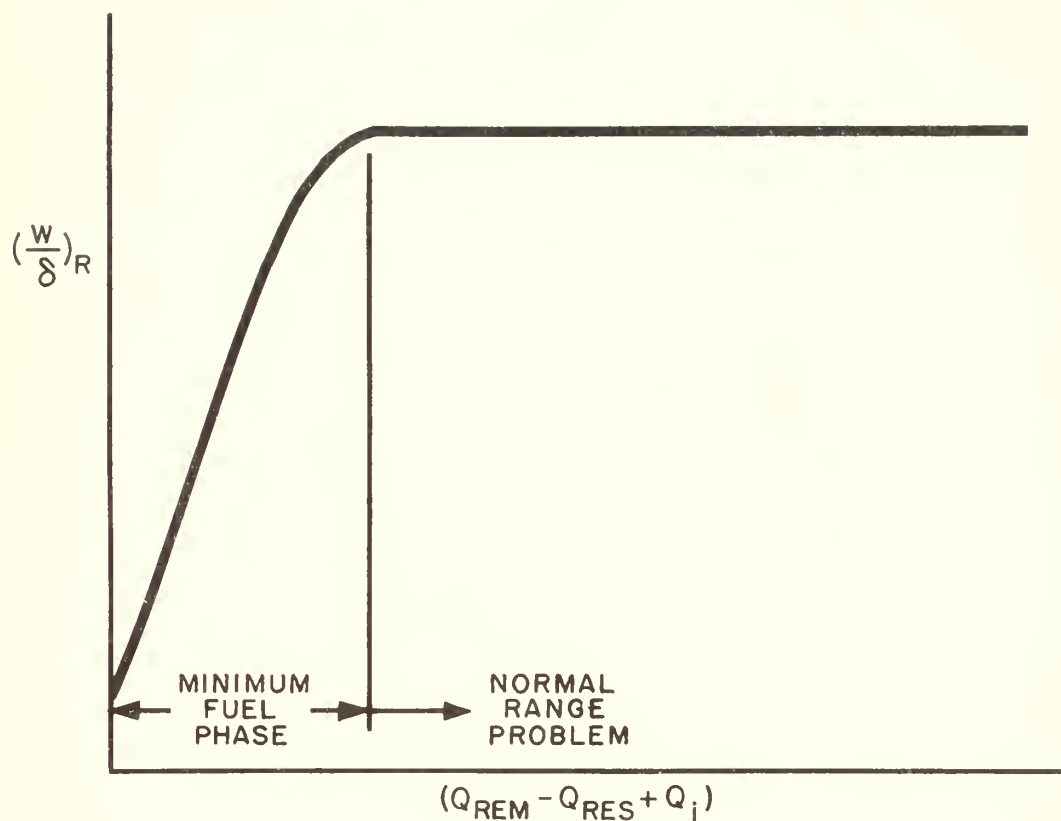


Fig. 13-1 Typical curve of  $(w/\delta)_R$  as a function of  $(Q_{REM} - Q_{RES} + Q_i)$ .





## CHAPTER 14

### INSTRUMENTATION PROBLEMS

In this chapter are presented some of the more important instrumentation problems encountered in this instrument.

#### I Use Of Metered Data

When flying according to the indicated value of  $\frac{W}{\delta}$  and  $M_R$  in the cruise phase of the flight it would be possible to obtain and use in the instrument the actual value of the specific range parameter by:

$$(SR \times \delta) = \frac{M \times \text{Constant} \times \sqrt{\theta}}{w_f} \times \delta \quad (14-1)$$

However, it is believed that using this value of  $(SR \times \delta)$  in the computer would cause excessive fluctuation of the range indication particularly when the pilot made throttle adjustments. It is considered more practical to use the actual values of  $W$  and  $\delta$  to obtain the instantaneous value of  $(SR \times \delta)_R$  through the use of the  $(SR \times \delta)_R$  vs  $\frac{W}{\delta}$  curve.

A suggested addition to the instrument is to provide means for adjusting the  $(SR \times \delta)_R$  vs  $\frac{W}{\delta}$  curve. This adjustment could be made by comparing under steady state cruise conditions the value of the  $(SR \times \delta)_R$  computed by the instrument with the measured value determined by equation (14-1). Then the  $(SR \times \delta)_R$  vs  $\frac{W}{\delta}$  curve in the instrument could be shifted up or down until the two values of  $(SR \times \delta)_R$  agreed. Means for making this comparison could be provided in the instrument or by a plug in unit carried



only when it is desired to check the  $(SR \times \delta)_R$  adjustment. This type of adjustment could be carried out periodically and would adjust the instrument to compensate for slight changes in the performance of the engine and minor changes in the aircraft configuration. A similar adjustment could be made to the endurance parameter vs  $\frac{W}{\delta}$  curve instrumented.

It should be emphasized that such an adjustment is not valid unless accurate measurements of  $w_f$ ,  $\theta$  and  $\delta$  can be obtained.

## II Acceleration Problems

The problems involved with aircraft acceleration in the proposed instrument were not investigated. However, such problems should be investigated before such an instrument is assembled. One of the principle acceleration problems is that of attaining the optimum climbing speed. The instrument as proposed assumes that when the climb button is depressed the aircraft is at or near the best climbing speed. Thus, the effect of being at a different speed must be examined. The results of an investigation of the acceleration problems may indicate that a correction factor should be built into the instrument, this factor being a function of the difference between the aircraft's present speed and the best climbing speed. The results may just dictate what acceleration procedure should be followed in attaining climbing speed. Or, the results may indicate the need for both a correction factor and a definite acceleration program.

The other principle acceleration problem is that of accelerating from the best climbing speed to the best cruising speed. It is believed that in this case all that will be required is to establish a set procedure to be followed; this procedure would not be instrumented but would be contained in the instructions as a recommended flight technique.

The acceleration problem from cruising speed to descent speed is not as important for there is a definite power setting

(idle) established for the descent.

### III Fuel Flow And Fuel Quantity Measurement<sup>(4)</sup>

One of the most important input quantities required for the instrument to function is that of fuel remaining. To obtain this quantity accurately requires either accurate measurement of fuel flow or fuel quantity or both. In the case of the aircraft with a main fuselage cell only it may be possible to measure fuel quantity. However, this is extremely difficult because of the construction features of the present day self-sealing fuel cells. Where some of the fuel is carried in wing tanks and/or external tanks a fuel flow meter appears to be the only suitable means for determining the fuel remaining in these cells. The accuracy of present day aircraft fuel quantity and fuel flow measuring devices appears at best only marginal for the requirements of the proposed instrument. Therefore, in view of the difficulties in measuring fuel quantity and the need of fuel flow information to determine the fuel remaining in wing and external tanks, it is believed that the best solution to the problem of measuring fuel quantity remaining is the development of an accurate fuel flow or mass flow meter and an associated integrator.

### IV Use Of Logarithmic Potentiometers

The large number of multiplications necessary in sections of the computer suggests the possibility of storing functions on logarithmic potentiometers, and multiplying by means of adding voltages proportional to their logarithms. A good example would be in the instrumentation of equation (7-4) for obtaining range in the cruising phase. The method of obtaining the value of  $(\frac{W}{\delta})_R$  as described in Chapter 5, using the stored temperature-pressure relations, is most easily obtained in logarithmic form, and may be subsequently used in equation (7-4) without modification.

The use of potentiometers ganged on a single shaft is also contemplated. One application of this is the use of the

"CONFIGURATION SET" knob to introduce both the weight and the drag corrections corresponding to a change in configuration.

#### V Predicted Temperature Inputs

The inputs of tropopause altitude and stratosphere temperature to the cruise ceiling computer are handled much more simply than might be expected, since they both enter a simple mechanical differential and appear as an input angle on one logarithmic potentiometer, as shown in Figure 5-3.

## CHAPTER 15

### CONCLUSIONS AND RECOMMENDATIONS

#### Part 1

As the performance of modern jet aircraft has improved, the penalties associated with operation of these aircraft at other than optimum flight speeds and altitudes have increased even more rapidly. The need exists, therefore, for a cruise control instrument which will indicate to the pilot the optimum manner in which the aircraft should be flown to realize the maximum benefits from the available fuel for a selected mode (either maximum range or maximum endurance), and display the predicted range and endurance figures based upon the available fuel and intended mode of operation.

The design of such an instrument appears feasible, although the number of variable factors affecting cruising performance prevent any simple solution. In order to handle these variable factors, and still meet the size and weight requirements for airborne equipment, extensive use of miniaturized components must be employed. In addition, all but the most necessary features must be eliminated, and considerable ingenuity in combining functions within the computer must be exercised.

Ambient temperature, fuel quantity and fuel flow inputs to the instrument are required to a high degree of accuracy. Accuracy of available fuel quantity and fuel flow measurement instruments which might be used in conjunction with the cruise control instrument is marginal, and further development of measurement instruments of this type is indicated. Present

instruments for measuring ambient temperatures in high speed flight are inadequate, from an accuracy standpoint. The possibility of simultaneously measuring ambient temperature and aircraft Mach number, as outlined in Appendix A, appears promising for cruise control and other purposes.

The numerous multiplications appearing in sections of the computer seem to indicate the desirability of using logarithmic potentiometers to store various functions, and then adding the output voltages of these potentiometer.

The method of storing predicted atmospheric data on temperature ratio vs pressure ratio is one of the features of this instrument. The accuracy of all output values, such as cruise ceiling, range and endurance predictions before reaching the cruising altitude will, in turn, depend on the accuracy of this stored atmospheric data.

This investigation did not cover acceleration effects on overall cruise performance. It is believed that under certain conditions these effects might play an important role. For example, the amount of fuel required to accelerate from a slow speed at low altitude to the speed required to enter a climbing airspeed schedule may be enough to affect the performance considerably. Further investigation along these lines appears justified.

## CHAPTER 15 (cont.)

### Part II

Pages 99 through 101 are **CONFIDENTIAL** and appear in a separately bound supplement to this thesis.

Appendix B is also included in this supplement.





## APPENDIX A

### AMBIENT TEMPERATURE INPUT TO THE CRUISE CONTROL INSTRUMENT

#### I Requirement

The necessity for accurate ambient temperature inputs for computing purposes in the Cruise Control instrument cannot be over emphasized. In the case of turbojet aircraft with thrust-limited cruise characteristics the range and endurance is affected to a marked degree by temperature since small temperature increases tend to depress the cruise ceiling considerably. A reasonable temperature tolerance can be determined by examining the curve of  $-\frac{W}{\delta}$  vs  $\theta$  along the cruise ceiling. A representative value for this tolerance for the turbojet aircraft with thrust-limited cruise characteristics is of the order of  $\pm 3^{\circ}\text{C}$

In the proposed cruise control instrument the problems associated with the ambient temperature input may be resolved into the following two phases:

#### 1. The Prediction Phase

This is the phase before the cruise altitude is reached. In this phase some sort of prediction must be made of upper altitude temperatures. This temperature prediction would be used in the computing components of the instrument for determining the displayed range and endurance.

This prediction requirement must be met with some form of stored data and obviously cannot be



met with a direct temperature measurement.

## 2. The Cruise Phase

This is the phase where the actual altitude is the cruise altitude. In this phase the stored temperature data could be used or the ambient temperature could be measured directly. Direct measurement would be the preferable source of the ambient temperature input to the computer providing an accuracy limitation of the order of  $\pm 2$  to 3 degrees Centegrade can be obtained.

## II Methods Of Storing Predicted Temperature Data

Two methods of storing this predicted temperature data will be considered:

- 1 The NACA standard day shifted by the amount of the sea level temperature variation from standard. While this method may be adequate for mid-latitudes and days during which the temperature lapse rate near sea level is constant and equal to the NACA standard value, the limitations of this method are considerable. Use of this method is considered inadequate, because of the strong dependence of the optimum cruising value of  $\frac{W}{\delta}$  on ambient temperature
- 2 The use of data based on an isothermal layer temperature setting, in conjunction with a tropopause altitude setting. In effect, these two settings determine the break point of a modified temperature curve. The lapse rate assumed below the break point is the same as the NACA Standard value of  $2^{\circ}\text{C}/1000\text{ ft}$ . The two values required to use this method might be obtained from

- a Current meteorological data, available at most air bases and aircraft carriers, obtained from meteorological balloons. This method should be fairly accurate, with temperature variations on the order of  $\pm 2$  to 3 degrees centigrade.
- b **Estimates**, based upon statistical studies. These studies show that the ten and ninety percentile lines generally fall within  $\pm 6^{\circ}\text{C}$  of the mean temperature value for a particular location and month.<sup>(6)</sup> The mean isothermal layer temperature might be determined by means of a table based on statistical data for different latitudes, time of year, locations (over land or sea mass) etc. This source of data is certainly not as accurate as that derived from current meteorological data but should still give much better results than method (1).

A comparison of methods (1) and (2) is shown in Figures A-1 and A-2.<sup>(7)</sup> It may be readily seen that the method described in (2) above gives a much better approximation to the existing curve of temperature vs. altitude at high- and low-, as well as mid-latitudes. It is an interesting fact that the average temperature in the isothermal layer over the polar regions is  $30^{\circ}\text{C}$  warmer than the average temperature of this layer over equatorial regions.

### III Methods Of Measuring Ambient Temperature Directly

Following are two possible methods which might be employed to get a direct measure from a fast moving aircraft of the ambient temperature.

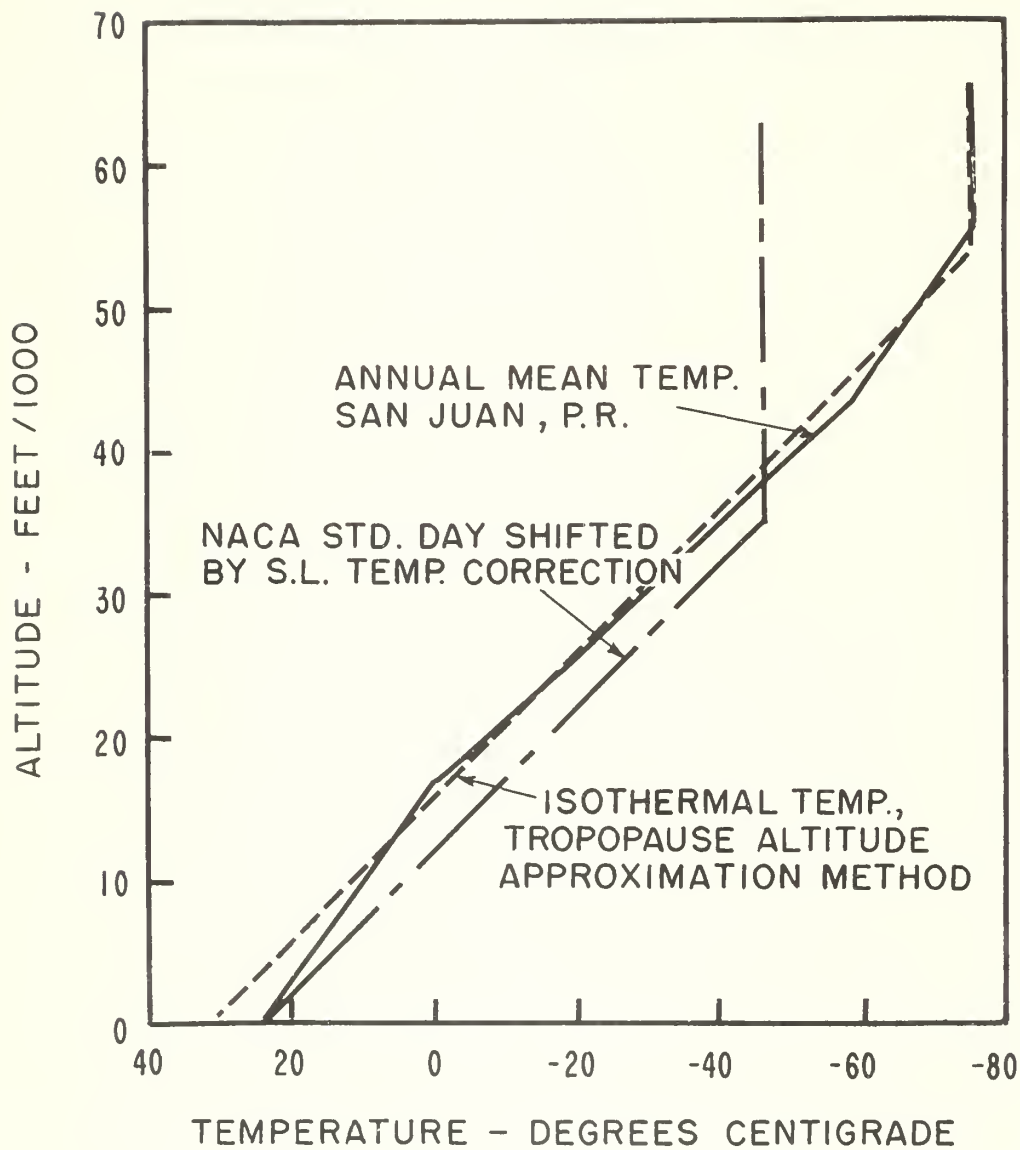


Fig. A-1 Two methods of approximation to mean temperature vs. altitude at San Juan, P.R.

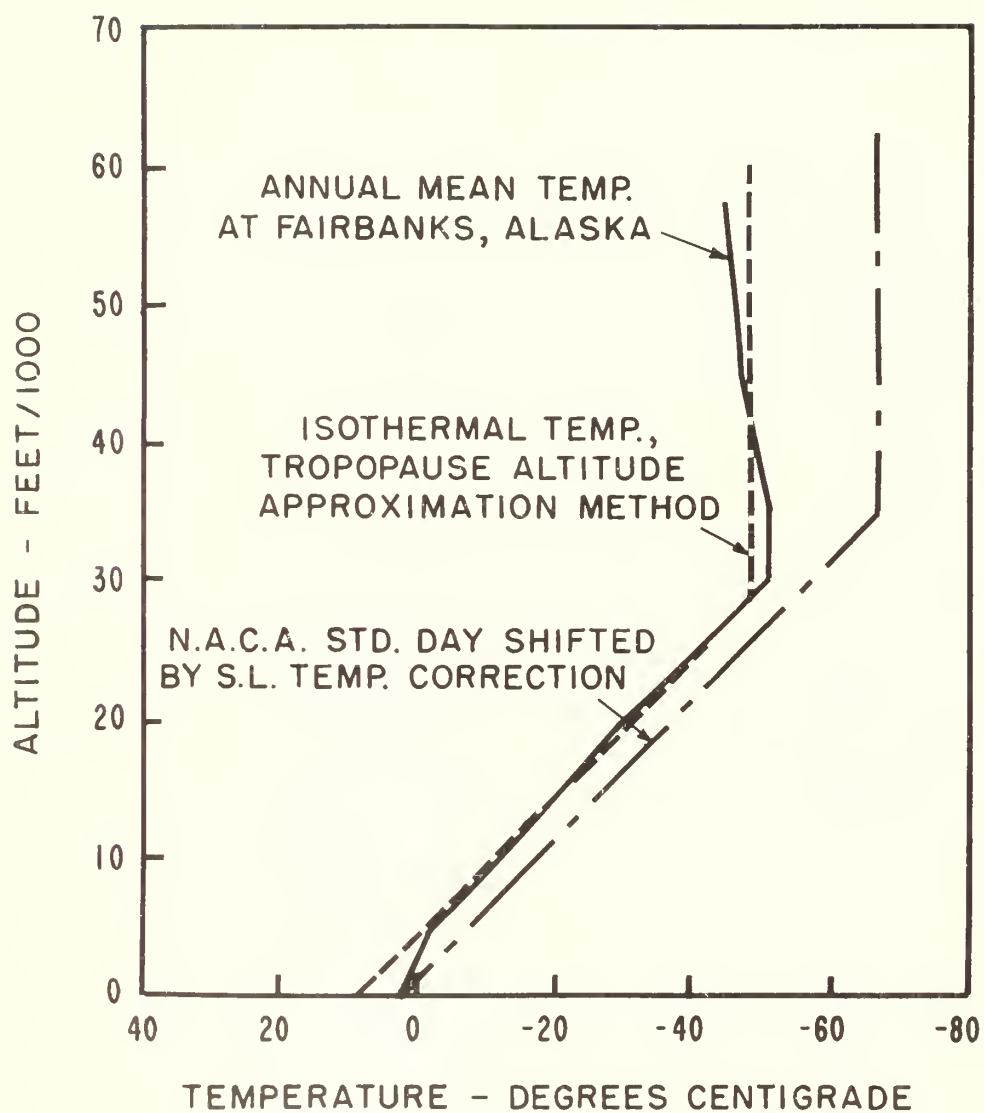


Fig. A-2. Two methods of approximation to mean temperature vs. altitude at Fairbanks, Alaska.

1. A method that could be employed at high subsonic speeds is outlined in Report No. A.A.E.E. /Tech/111 Ministry of Supply Aeroplane and Armament experimental Establishment Boscombe Down (British). A knife edge bulb thermometer, A and AEE cylindrical bulb thermometer or a Meteorological Office (British) impact bulb thermometer could be employed providing accurate recovery factors, K, can be obtained for these instruments. Then the following formulae could be employed.

$$\begin{aligned}
 T_i &= T_a + K \left( \frac{V}{86.8} \right)^2 \\
 &= T_a + \left( \frac{V_1}{86.8} \right)^2 \frac{K}{\text{actual relative density}} \\
 &= T_a \left\{ 1 + \frac{K}{288p} \left( \frac{V_i}{86.8} \right)^2 \right\}
 \end{aligned}$$

$$\text{then: } T_a (^{\circ}\text{K}) = \frac{T_i}{\left[ 1 + \frac{K}{288} \frac{1}{p} \left( \frac{V_i}{86.8} \right)^2 \right]}$$

where:

$T_a$  = ambient temperature

$T_i$  = indicated temperature

$p$  = ratio of static pressure to standard sea level pressure

$V$  = true air speed

$V_1$  = equivalent air speed

$K$  = recovery factor

The difficulty with this system is that the recovery factor is difficult to obtain and must be

measured to a high degree of accuracy. Also, the recovery factor,  $K$ , is not a constant with altitude. Thus, in order to employ these instruments a curve of  $K$  vs. altitude must be determined and incorporated in the instrument.

- 2 Another method of measuring the ambient temperature from a high speed aircraft has been suggested by Professor E. Mollo-Christensen of M. I. T., Cambridge, Mass. By this method the aircraft's Mach number and the speed of sound (hence ambient temperature) can be measured at high subsonic and supersonic aircraft velocities. The scheme for measuring these values at supersonic velocities will be outlined briefly to demonstrate the method.

The supersonic system requires a point sound source for generating a sharp sonic pip and a sound receiver at a known distance aft of the sound source. Both the sound source and the receiver would be mounted on a stinger projecting into the air stream ahead of the aircraft. The stinger would be as small in diameter as possible in order to reduce boundary layer interference. The time interval between the time the output signal leaves the sound source and the time the after portion of the generated spherical pressure wave is sensed by the receiver, and the time interval between the time the output signal leaves the sound source and the time the receiver senses the forward portion of the pressure wave, may be measured electronically. Knowing these two time intervals and the distance between the sound source and the receiver, both the aircraft's Mach number and the speed of sound (from which ambient temperature can be obtained) may be obtained as follows:

Referring to Figure A-3, the following relations may be derived. Subscript 0 refers to the outgoing sound pip, subscripts 1 and 2 refer to the sensing of the first and second pressure waves respectively.

$$d = x_1 (1 + \sin \theta) = x_1 (1 + \frac{1}{M})$$

$$d = x_2 (1 - \sin \theta) = x_2 (1 - \frac{1}{M})$$

$$\begin{aligned} t_1 - t_0 &= \frac{x_1 \sin \theta}{a_0} = \frac{x_1}{M a_0} \\ &= \frac{d}{(1 + \frac{1}{M}) M a_0} \\ &= \frac{d}{(M + 1) a_0} \end{aligned} \quad (A-1)$$

$$\begin{aligned} t_2 - t_0 &= \frac{x_2 \sin \theta}{a_0} = \frac{x_2}{M a_0} \\ &= \frac{d}{(1 - \frac{1}{M}) M a_0} \\ &= \frac{d}{(M - 1) a_0} \end{aligned} \quad (A-2)$$

From equations (A-1) and (A-2) the two unknowns  $M$  and  $a_0$  may be found and from  $a_0$  the absolute temperature may be determined.

A similar system employing two receivers, one forward and one aft of the sound source, could be employed to measure the same quantities at subsonic speeds by means of relationships similar to those used in the supersonic example.

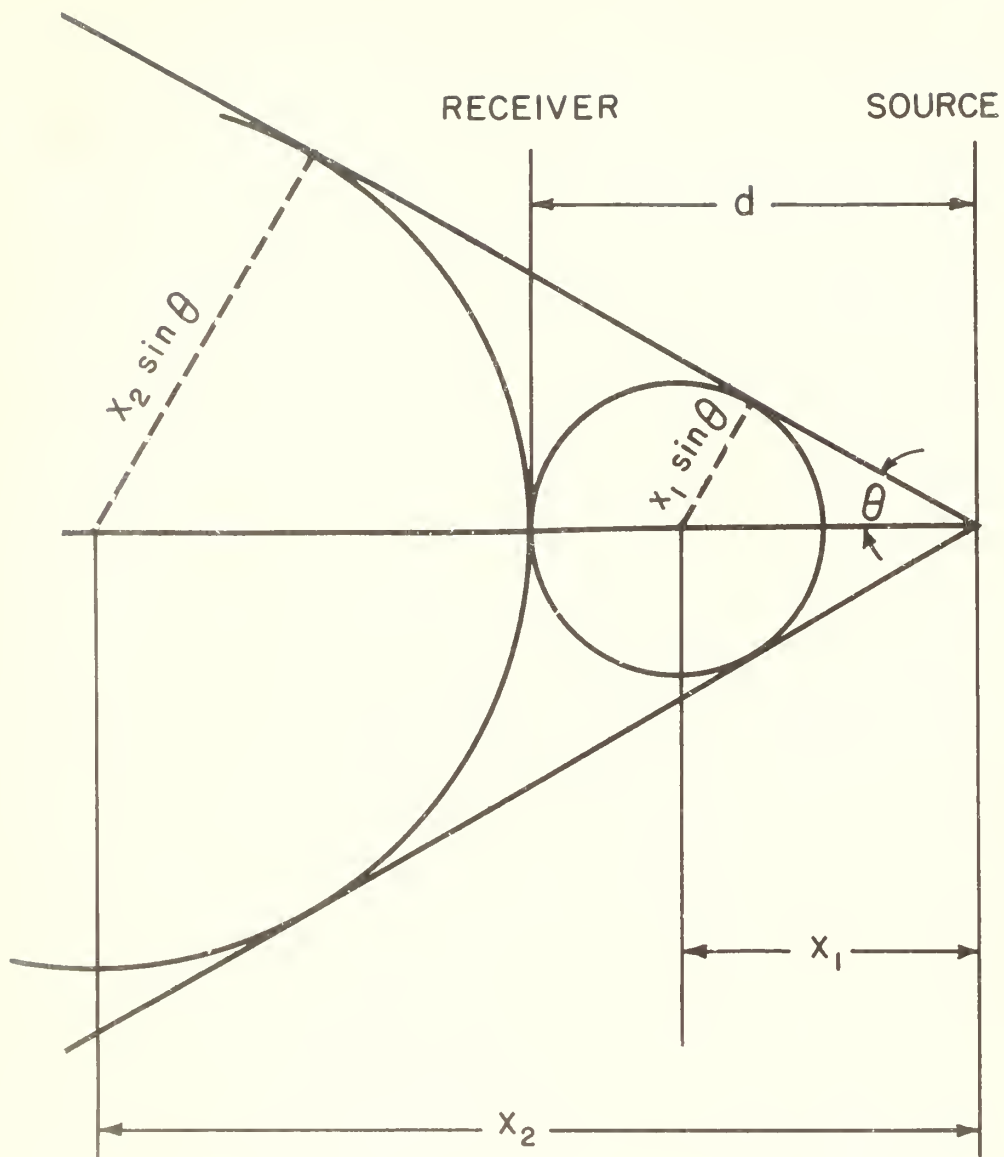


Fig. A-3. Scheme for measuring Mach number and speed of sound in supersonic flight.



#### IV Stored Temperature Instrumentation

Using the isothermal-temperature, tropopause-altitude method of representing existing atmospheric conditions, as described above, a relation between the pressure ratio,  $\delta$ , and the temperature ratio,  $\theta$ , may be derived.

A constant temperature lapse rate in the troposphere,  $k_T$ , equivalent to the NACA standard day may be written:<sup>(8)</sup>

$$k_T = 0.001983 \text{ } ^\circ\text{C}/\text{ft} \quad (\text{A-3})$$

The temperature at any altitude,  $h$ , in the troposphere may be written:

$$T = T_O - k_T h \quad (\text{A-4})$$

Since the temperature and altitude of the tropopause "break-point" are used to define the atmospheric conditions, equation (A-4) evaluated at the tropopause gives:

$$T_O = T_B + k_T h_B \quad (\text{A-5})$$

where the subscript B refers to the tropopause "break-point".

To obtain an expression for pressure as a function of altitude, consider the unit element of the atmosphere shown in Figure A-4. Then summing forces in the  $h$  direction shown in Figure A-4 gives:

$$p - (p + dp) - \rho g dh = 0 \quad (\text{A-6})$$

and therefore:

$$dp = - \rho g dh \quad (\text{A-7})$$

The equation of state gives:

$$\rho = \frac{p}{kT} \quad (\text{A-8})$$

in which  $T$  is in absolute degrees, so that:

$$\frac{dp}{p} = - g \frac{dh}{kT} \quad (\text{A-9})$$

$$\text{But } R = \frac{k}{g} = 53.3 \frac{\text{ft}}{\text{o}_F} = 95.94 \frac{\text{ft}}{\text{o}_C}, \text{ hence:} \quad (\text{A-10})$$

$$\frac{dp}{p} = - \frac{dh}{RT} \quad (\text{A-11})$$

Substituting values from equations (A-4) and (A-5) gives,

$$\frac{dp}{p} = - \frac{1}{R} \left[ \frac{dh}{T_B + k_T (h_B - h)} \right] \quad (\text{A-12})$$

In order to integrate, it should be noted that:

$$d [T_B + k_T (h_B - h)] = - k_T dh \quad (\text{A-13})$$

so that equation (A-12) may be written

$$\frac{dp}{p} = \frac{1}{Rk_T} \frac{d [T_B + k_T (h_B - h)]}{T_B + k_T (h_B - h)} \quad (\text{A-14})$$

To find the pressure  $p$  corresponding to the altitude  $h$ , the left side of equation (A-14) is integrated between  $p_o$  and  $p$  and the right side between  $o$  and  $h$ , thus:

$$\int_{p_o}^p \frac{dp}{p} = \frac{1}{Rk_T} \int_o^h \frac{d [T_B + k_T (h_B - h)]}{T_B + k_T (h_B - h)} \quad (\text{A-15})$$

which becomes:

$$\ln \frac{p}{p_o} = \ln \delta = \frac{1}{Rk_T} \ln \frac{[T_B + k_T (h_B - h)]}{T_B + k_T h_B} \quad (\text{A-16})$$

or alternately;

$$\ln \delta_{\text{atm}} = \frac{1}{Rk_T} \ln \frac{T}{T_B + k_T h_B} \quad (\text{A-17})$$

and dividing the numerator and denominator of the expression involving temperatures by  $T_{o_{\text{std}}}$ , results in

$$\ln \delta_{\text{atm}} = \frac{1}{Rk_T} \ln \frac{\theta}{\theta_B + \frac{k_T}{T_{o_{\text{std}}}} h_B} \quad (\text{A-18})$$

Equation (A-18) may be rewritten:

$$\ln \delta_{\text{atm}} = \frac{1}{Rk_T} \left[ \ln \theta - \ln \left( \theta_B + \frac{k_T}{T_{\text{Ostd}}} h_B \right) \right] \quad (\text{A-19})$$

Equation (A-19) may be instrumented using logarithmic potentiometers; thus, equation (A-19) represents the atmosphere in the troposphere. At altitudes above the tropopause,

$\theta = \theta_B = \text{constant}$  for a given pre-flight setting, so that a limit stop, or some similar method of limiting  $\theta$  is necessary so that it will not become smaller than  $\theta_B$ .

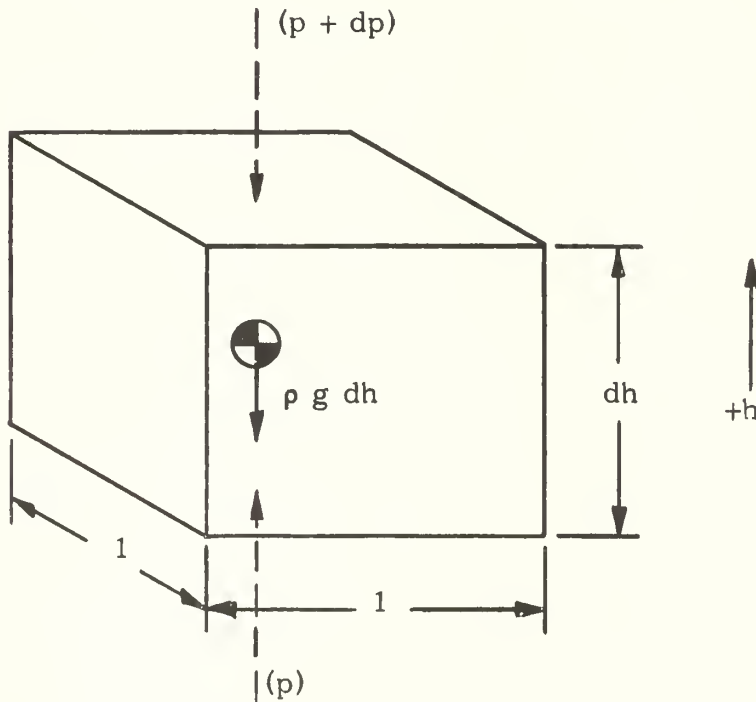


Fig. A-4 Unit Element Of Atmosphere

## APPENDIX B

### PERFORMANCE CURVES FOR THE F-100C

Pages 115 through 130 are CONFIDENTIAL and appear in a separately bound supplement to this thesis.

Part II of Chapter 15 is also included in this supplement.



## APPENDIX C

### TABLE OF SYMBOLS

$S$	- wing area
$\frac{R}{C}$	- Rate of Climb
$F$	- thrust Force
$D$	- Drag Force
$W$	- Weight
$V$	- Velocity
$\rho$	- air density
$T$	- Temperature
$p$	- pressure
$R$	- gas constant
$\delta$	- pressure ratio, $\frac{p}{p_{SL}}$
$\theta$	- temperature ratio, $\frac{T^{\circ}_{absolute}}{T_{SL}^{\circ}_{absolute}}$
$M$	- Mach No.
$k_{( )}$	- constant
$K_{( )}$	- constant
$C_{( )}$	- constant
$S_{[m;n]}$	- sensitivity where m = input , n = output
$SR$	- Specific Range
$w_f$	- fuel flow rate
$Q$	- fuel Quantity
$E$	- Endurance
$h$	- altitude

- Standard Day - NACA Standard Day
- Hot Day - The ambient temperature at each altitude is  $25^{\circ}\text{C}$  hotter than the NACA Standard Day
- Cold Day - The ambient temperature at each altitude is  $25^{\circ}\text{C}$  colder than the NACA Standard Day

## SUBSCRIPTS

- SL - Sea Level
- o - refers to start of military power climb, at sea level
- 1 - refers to start of cruising flight path
- 2 - refers to end of cruising flight path
- i - refers to initial point, in climb, at other altitude than sea level
- cc - climb cruise
- W - Wind
- R - value for maximum Range
- E - value for maximum Endurance
- c - configuration
- cr - cruise
- cl - climb
- lc - level cruise
- d - descent
- inst. - instantaneous
- atm. - atmosphere
- B - condition at tropopause break point
- rem. - remaining

res. - reserve  
ref. - reference value  
mf - minimum fuel  
T - Temperature





## APPENDIX D

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